# PIPING VIBRATION AND STRESS

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## INTRODUCTION

To even a casual observer, a most obvious effect of pulsations is that it forces piping and other plant systems into sustained vibrations and, under some conditions, the vibrations can cause fatigue failures at critical, high bending stress regions in the mechanical systems. The existence of such pulsation-induced mechanical vibrations suggest two obvious approaches to control and one approach which is perhaps less obvious. These are:

1. Supply mechanical restraints which will prevent movement of the pipe.

2. Eliminate or control the pulsations.

3. Eliminate the coupling of pulsations as forces into the piping.

While each of these approaches are valuable, no one approach is optimum in all cases and any one by itself can prove excessively expensive. The cost of mechanically restraining compressor piping or overhead plant piping, for example, soon causes the engineer to seek help from other control approaches. A similar situation exists with pulsation control. If pulsation suppressors are designed to eliminate "all" pulsations (i.e., to a level that any piping system could be utilized) then it is soon found that pressure vessels of excessive size are required.

The concept of decoupling the pulsations from forcing the mechanical system into vibration will be discussed in a later section, but basically it involves controlling the location of bends, constrictions and piping discontinuities relative to the pulsation standing waves. It is difficult, for example, to excite an infinitely long, straight, constant diameter pipe into vibration from internal pulsations. In more realistic piping configurations, however, there are also things that can be done to minimize pulsation shaking forces. These, too, will be described later in this chapter.

While each of the above approaches are useful in controlling known piping vibration problems, two fundamental questions remain which can drastically reduce the time and effort involved in field fixes:

1. Are you aure the vibrations are excessive and require reduction?

2. What can you do at the design stage to prevent the problem?

Again, these will be dealt with in subsequent sections involving "Criteria", "Field Testing" and "Simulation Techniques for Predicting Pulsation Induced Vibrations".

## PIPING VIBRATION AND STRESS CRITERIA

One of the major reasons why pulsation control alone should not be used to control flow-induced piping vibrations lies in the fact that there are no pulsation criteria which can be reliably used for preventing vibrations. In spite of the fact that many such criteria have been evolved, it is not pulsations per se which are the problem, but rather the dynamic stress levels which result in the pipe wall. Whenever vibratory stress exceeds the endurance level of the material, piping failure is imminent.

By similar argument, it can be seen that vibration amplitude criteria for piping systems are likewise dangerous and, again, are fundamentally the wrong approach unless consideration is given to the configuration and dimensions of the piping system being considered. The technical literature is replete with vibration criteria for plant piping, machinery, and structural systems which specify "allowable" vibration amplitudes as a function of frequency as shown in Figure l. Such criteria are based largely upon the experience of field personnel operating or working with such equipment. While they may be applicable in a statistical sense to average or typical piping, they are fundamentally incorrect because they do not consider the configuration involved. As such, they introduce considerable risk when used in evaluating any specific piping system as they may result in a degree of design confidence which is unwarranted by the design procedure used. Although the criteria are based on typical or average conditions, they do not normally contain such a warning or supply a definition of the limits of what constitutes average or typical.

The problem with any such criterion is not so much that it is not applicable to many plant systems but rather the cost of failure and downtime in those cases in which it does not work. While the statistical data from which the criterion was generated proves it works in most cases, the risk that it may not work for the next design should often dictate a more thorough analysis. Note that the criterion as presented does not differentiate between a stiff compressor manifold system and a flexible scrubber lead line. If the criterion is sufficiently conservative to protect the compressor manifold system, it will normally be overly conservative for the lead line. It should also be noted that the stress level in a pipe is a function of physical distortion only (i.e., strain), and is not a function of frequency for the general case. If frequency is to be one of the controllable allowables in vibration amplitude, it must include cognizance of the type of piping span involved and its resonant frequency and mode shape, as discussed below.

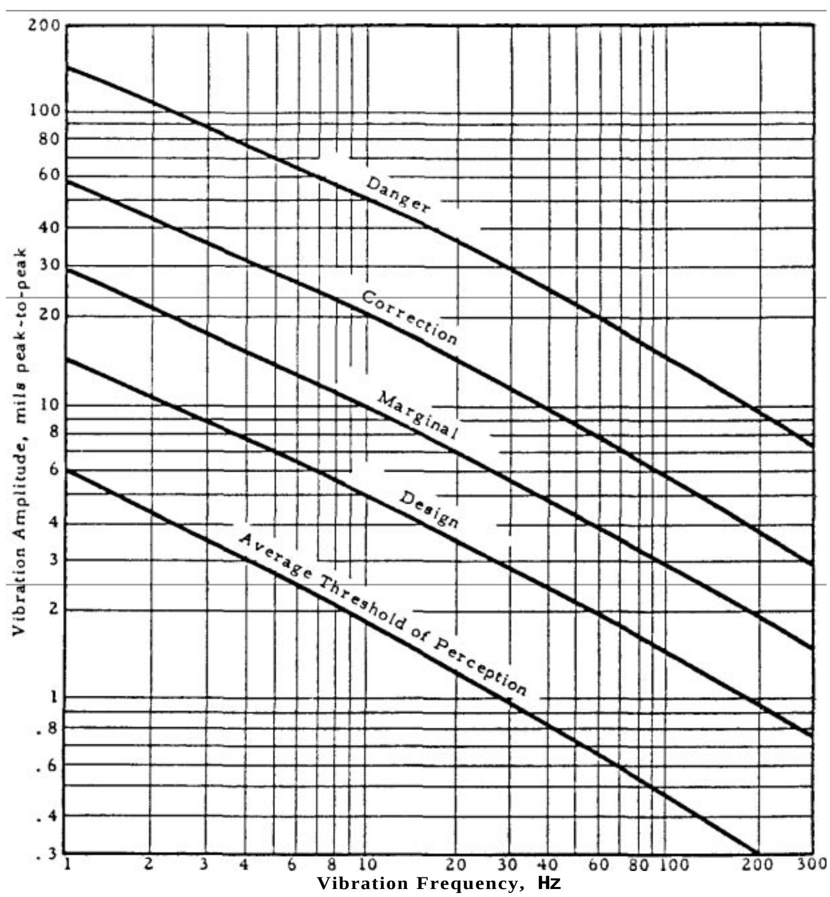



Figure 1. Allowable Piping Vibration Levels

Note: Indicated vibration limits are for average piping system constructed in accordance with good engineering practices. Make additional allowances for critical applications, unreinforced branch connections, etc.

## Stress Predictions in Idealized Pipe Spans

For any span configuration, it is feasible to calculate the stress which results from a given deflection (stress per mil, or s/y), providing the end conditions and vibratory mode shape (deflection profile) are known.

The general equation relating maximum stress in a pipe to the maximum deflection along the span is given below for the lowest vibratory resonance mode (note that the maximum stress and maximum deflection are generally not at the same point):

$$s/y = \frac{ED}{2} \; \frac{\ddot{y}}{y} \; \frac{\lambda}{\ell^2}$$

Where:

- s = Stress at maximum stress point, psi ($lb/in^2$)
- E = Elastic Modulus, $lb/in^2$
- D = Diameter, in
- $\ddot{y}$ = $d^2y/dx^2$ evaluated at maximum stress point
- y = Deflection, mils, at maximum deflection point
- $\ell$ = Span length, in
- L = Span length, ft
- $\lambda$ = Frequency factor

for steel pipe, this becomes

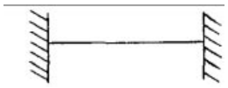
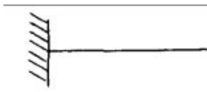
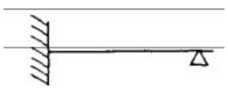
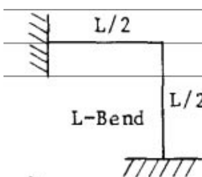
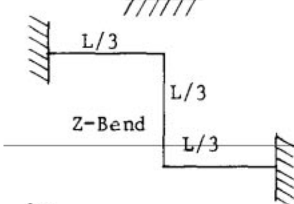
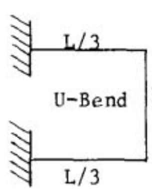
$$s/y = \frac{D}{L^2}\left[\frac{30 \times 10^6}{2(144)\;1000}\right]\frac{\lambda\ddot{y}}{y}$$

$$= 104.17 \; \frac{D}{L^2} \; \frac{\lambda\ddot{y}}{y} \text{ psi stress/mil deflection}$$

Solution of this equation for several span configurations is given in Table 1. This table can be used either:

1. To determine the stress resulting from a given deflection (at the maximum deflection point) or
2. To establish maximum allowable deflections.

Table 1. Constant Factors for Calculating the Stress Per Mil (s/y) in Various Pipe Spans

| Beam Type | |
|---|---|
|  Fixed-Fixed | $s/y = 2935 \frac{D}{L^2}$ |
|  Cantilever | $s/y = 366.3 \frac{D}{L^2}$ |
|  Simply Supported | $s/y = 1028 \frac{D}{L^2}$ |
|  Fixed/Simply Supported | $s/y = 2128 \frac{D}{L^2}$ |
|   L-Bend | $s/y = 1731 \frac{D}{L^2}$ |
|   Z-Bend | $s/y = 3600 \frac{D}{L^2}$ |
|   U-Bend | $s/y = 2836 \frac{D}{L^2}$ |

## Non Ideal Beams

Table 1 assumes idealized end conditions. As described in a later section, a typical straight continuous span with strap and pier supports most nearly matches the resonant frequency prediction of the fixed/simply supported beam (A = 15.42). In field situations where the lowest resonant frequency of the span can be measured (as by bumping with a cross-tie), the stress per mil can be adjusted to compensate for nonideal supports by the following equation (for straight spans only):

$$(s/y)_{actual} = (s/y)_{calculated} \left( \frac{f_o \text{ measured}}{f_o \text{ calculated}} \right) \times SCF$$

Where:

$f_o$ = resonant frequency

SCF = stress concentration factor, as may be applicable to the point (fitting, etc.) where maximum stress occurs.

For ideal simply supported spans, the above linear relationship between stress and frequency may be as much as 50% high (conservative), but for other end conditions accuracy is generally within about 5%.

## Vibration Criteria

It was noted in the first section above, that generalized piping vibration criteria are fundamentally incorrect unless configurations and dimensions are included. This section will therefore include these considerations and generate new vibration criteria, at least for some piping configurations.

API Standard 618, "Reciprocating Compressors for General Refinery Services", 2nd Edition, 1974, in Section 3.3.2.1.a., states that the vibration induced cyclic stresses should be less than 26,000 psi peak-to-peak for steel pipe below 700 F. This criterion is based upon the curve given in Figure 2, "Allowable Amplitude of Alternating Stress Intensity, Sa.", given in ANSI USA Standard B31.7, "Nuclear Power Piping" and other ASME codes. Extensive use of these curves has shown them to be conservative even when the combined steady state stresses introduced by pressure, thermal and weight loading are near the yield stress.

Based upon some 25 years of experience with piping vibration and failures, SwRI has developed vibration amplitude versus frequency criteria (Figure 1) in lieu of a more exact technique for estimating the vibratory dynamic stress in specific piping configurations. The disadvantage of criteria such as given in Figure 1 is that if they are conservative for stiff compressor manifold systems they can be overly conservative for long flexible lead lines.

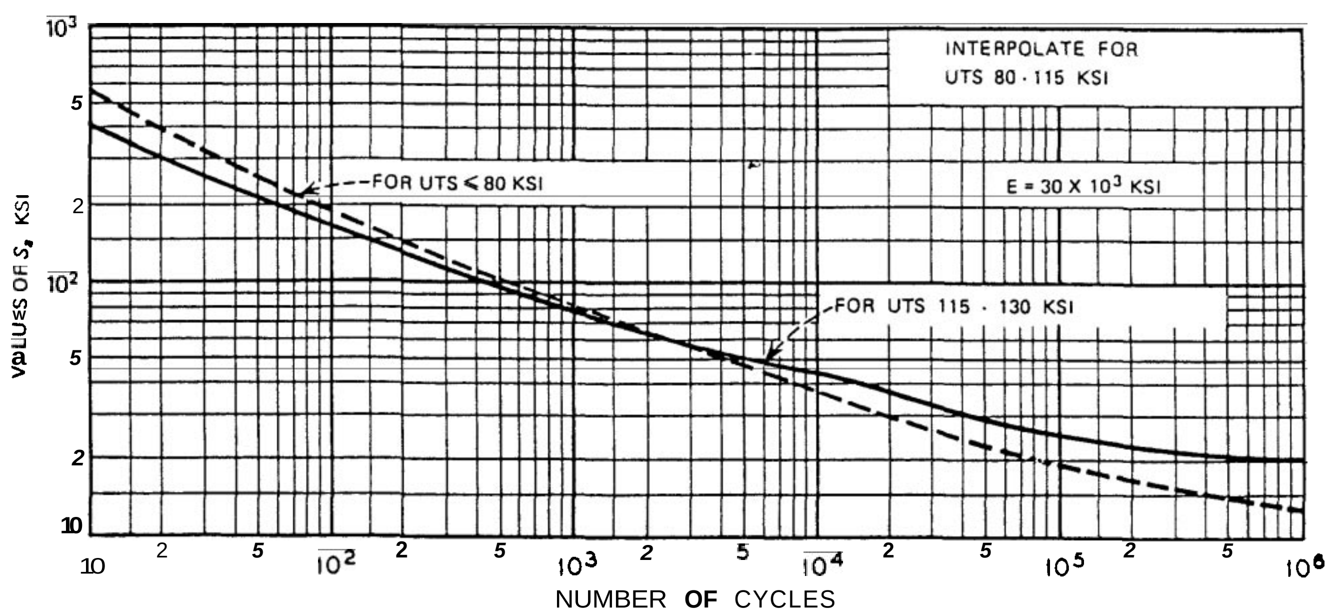



Figure 2. Allowable Amplitude of Alternating Stress Intensity, $S_a$, for Carbon and Alloy Steels With Metal Temperatures Not Exceeding 700°F

The importance of configuration is illustrated by comparing a cantilever pipe section with a fixed-fixed span or L-bend of equal length. Obviously, the stress generated in the cantilever span due to a 1-inch end deflection is different than in that generated in the other configurations by an equal deflection. It is also apparent that a lower stress will be generated in a long span than in a short one. An investigation into the dynamics of such spans shows that the variation in stress per unit deflection tracks rather directly with resonant frequency for a given span type; i.e., a long flexible span has low stress per unit deflection and low resonant frequency. This frequency variation may be used advantageously to normalize (non-dimensionalize) allowable stress criteria. For example, the usual allowable deflection vs. frequency plots could be made substantially more accurate if the abscissa were changed from vibration frequency to fundamental span resonant frequency.

The vibration allowable deflection criteria for L-bend piping spans is given in Figure 3 for first and second mode resonant vibrations. Note that when the usual displacement criterion is multiplied by frequency, an almost flat, horizontal criterion curve results and the product of vibration amplitude and frequency is, of course, vibrational velocity. The approach used in developing these criteria follow the analysis procedure described in the

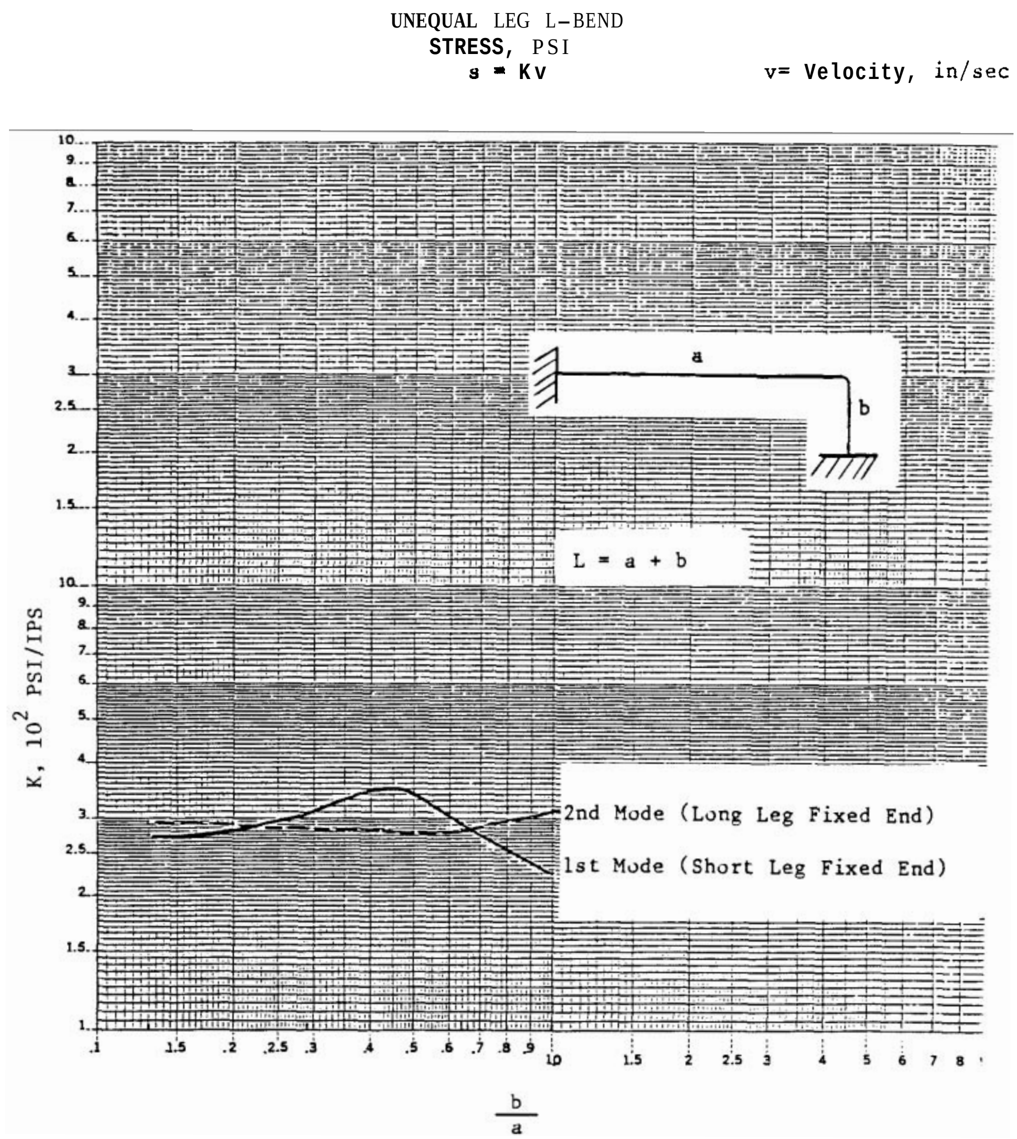



Figure 3. Allowable Deflection Criterion for Ell Bends, For First and Second Mode Resonant Vibrations (Steel Pipe)

preceding section, wherein the deflection required to produce 13,000 psi bending stress is used as the standards of acceptability (i.e., when stress equals the endurance limit of the steel.)

Similar criterion curves are now being generated for other piping span configurations as a part of the SGA Research Program, and a nomograph is being prepared to compute stress as a function of deflection for a broad spectrum of span configurations.

## DEVELOPMENT OF VIBRATION AMPLITUDE AND VELOCITY CRITERIA

The natural frequency of a uniform beam can be calculated by any of the following equations:

$$f = \frac{\lambda}{2\pi} \sqrt{\frac{gEI}{\mu \ell^4}}$$

$$f = \frac{\lambda}{2\pi} \sqrt{\frac{gE}{\gamma \ell^4} \left(\frac{I}{A}\right)}$$

$$f = \frac{\lambda k}{2\pi \ell^2} \sqrt{\frac{gE}{\gamma}}$$

Where:

$\mu = \gamma A$

$\gamma$ = Density lb/in$^3$

A = Metal area, in$^2$

$k = \sqrt{\frac{I}{A}} \approx 0.34\ D$ for pipe (see Figure 4)

Using the expression of 0.34 for the radius of gyration,

$$f = \frac{\lambda}{2\pi} \sqrt{\frac{gE}{\gamma}} \left(\frac{0.34D}{\ell^2}\right)$$

(Note: $\ell$ is in inches.)

For steel pipe this becomes:

$f = 76 \frac{\lambda D}{L^2}$, if L is in feet.

Solving for $D/L^2$

$$\frac{D}{L^2} = \frac{f}{76\lambda}$$

Substituting this into the stress per mil equation:

$$s/y = 104.17 \frac{D}{L^2}\lambda \frac{\ddot{y}}{y}$$

$$s/y = 104.17 \frac{f}{76}\lambda \frac{\ddot{y}}{y}$$

$$s/y = 1.371\ f \frac{\ddot{y}}{y}$$

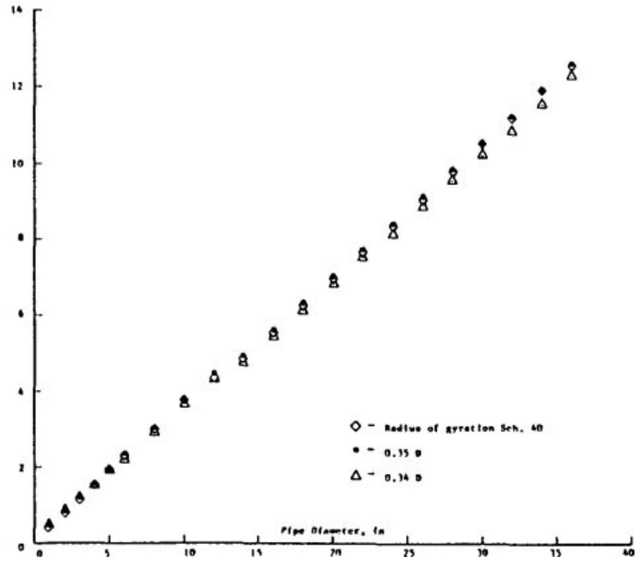



Figure 4. Comparison of Approximations for Radius of Gyration Versus True Value for Various Pipe Sizes

For sinusoidal vibrations, vibrational velocity (v) is $2\pi f y$, and the stress equation can be written in terms of stress per unit of vibrational velocity:

$$\frac{s}{2\pi f y} = \frac{1000}{2\pi} (1.371) \frac{\ddot{y}}{y}$$

$$\frac{s}{v} = 218.2 \frac{\ddot{y}}{y}$$

In another form,

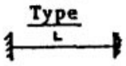
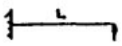
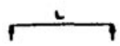
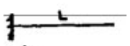
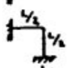
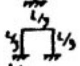
$$\frac{s}{Y} = K' \frac{D}{L^2}; \text{ where } K' = \frac{K}{144 \times 10^5}$$

$$\frac{s}{v} = 1.46 \times 10^{-5} \frac{K}{\lambda}$$

$$\frac{s}{v} = 2.1 \frac{K'}{\lambda}$$

Table 2 gives the allowable stress per velocity for straight beams and equal leg bends vibrating at their lowest resonant frequency. Note that the range of stress per velocity only ranges from 218 to 370 psi/ips. If the fixed end stress coefficients are used (275 psi/ips) then this value would be within 30 percent for the

Table 2. Summary of Stress Factors and Allowable Velocity

| Beam Type | Frequency Factor $\lambda$ | Streee Coefficient $K'$ | $2.1 \frac{K'}{\lambda}$ psi/ips | Vall ipe, 0-p 13,000 psi | Vall SCF=4 |
|---|---|---|---|---|---|
|  | 22.4 | 2935 | 275 | 47.3 | 11.8 |
|  | 15.4 | 2128 | 290 | 44.8 | 11.2 |
|  | 9.87 | 1028 | 218 | 59.6 | 14.9 |
|  | 3.52 | 366 | 218 | 59.6 | 14.9 |
|  | 15.0 | 1731 | 242 | 53.7 | 13.4 |
|  | 18.0 | 2836 | 332 | 39.2 | 9.8 |
|  | 20.4 | 3600 | 370 | 35.1 | 8.8 |

minimum and maximum values. If the stress per velocity is equated to the maximum allowable dynamic stress ($S_{all}$ = 13,000 psi 0-p), then the allowable velocity (Vall) is obtained from:

$$V_{all} = \frac{S_{all}}{(S/V)_{all}} = \frac{13,000}{275} = 47.3 \text{ ips}$$

## Configurational Corrections

In order to apply the criteria to a real piping system, the stress concentration factor and other reduction factors such as correction for concentrated weights, non-ideal end conditions, changes in pipe diameters and effect of vibration mode shape must be taken into consideration.

$$V_{all} = \frac{S_{all}}{(S/V)_{all}\ C_1\ C_2\ C_3\ C_4\ C_5}$$

$C_1$ = Correction factor to compensate for the effect of concentrated weights along the span of the pipe.

$C_2$ = Stress Concentration Factor.

$C_3$ = A correction factor accounting for pipe contents and insulation.

$$= \left(1.0 + \frac{W_F}{W} + \frac{W_{ins}}{W}\right)^{1/2}$$

$W_F$ = Weight of pipe contents per unit length.

$W$ = Weight of pipe per unit length.

$W_{ins}$ = Weight of pipe insulation per unit length.

$C_4$ = Correction factor for end condition different from fixed ends and for configurations different from straight spans:

$C_4$ = 1 for straight spans fixed at both ends.

= 0.75 for cantilever and simply supported beams.

= 1.35 for equal leg Z-bend.

= 1.2 for equal leg U-bend.

$C_5$ = Correction factor to compensate for vibration mode shapes other than the first.

Based upon SwRI experience, a stress concentration of 4 is appropriate for welds in branch connection without being overly conservative. Plots of the correction factor $C_1$ are given in Figure 5. In most cases the contents and insulation weight is less than the pipe weight itself so $C_3$ is generally less than 1.5.

Applying the stress concentration, the allowable velocity becomes:

$$V_{all} = \frac{13,000}{275(4)} = 12 \text{ ips}$$

This would apply to uninsulated pipe vibrating at resonance in its fundamental mode.

If the maximum effect of concentrated weights (which is approximately 8), pipe contents, and a safety factor of 2 are used:

$$V_{all} = \frac{12}{1.5(8)(2)} = 0.5 \text{ ips}$$

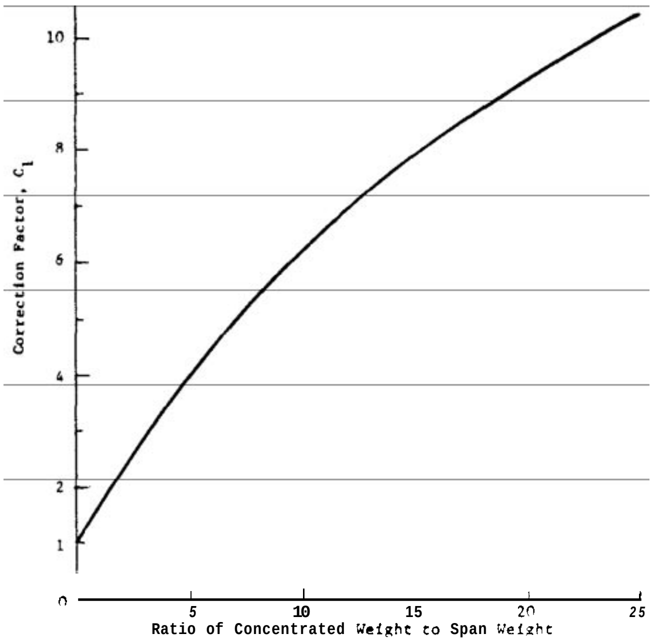



Figure 5. Correction Factor, $C_1$

A comparison of these criteria with ones previously developed is given in Figure 6.

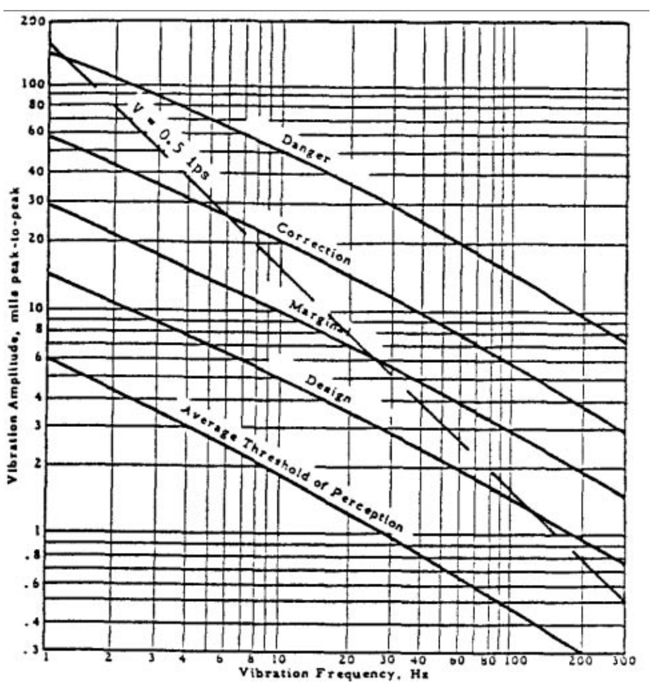
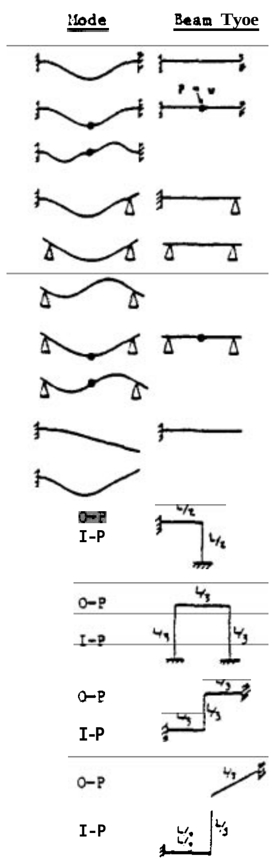



Figure 6. Allowable Piping Vibration Levels with Velocity Criteria

Higher Mode Vibrations

A tabulation of the stress coefficients for higher modes and for some straight beams with concentrated weights equal to the pipe span weight are given in Table 3.

Table 3. Higher Mode Stress Coefficients and Deflection/Stress Relationships for Various Span Configurations
[Where: Concentrated Weight (P) = Pipe Weight (w)]




| Mode | K' | K'/λ | S/Δ | S/V |
|---|---|---|---|---|
| | 2927 | 13 | 129 | 275 |
| | 2615 | 221 | 696 | 463 |
| | 8336 | 141 | 222 | 295 |
| | 2125 | 138 | 93.5 | 289 |
| | 1029 | 104 | 27.4 | 219 |
| | 4155 | 105 | 111 | 222 |
| | 1180 | 207 | 31.4 | 434 |
| | 4137 | 108 | 110 | 225 |
| | 389 | 110 | 11.6 | 232 |
| | 2292 | 105 | 68.4 | 220 |
| O-P | 1744 | 116 | 37.6 | 226 |
| I-P | 8696 | 145 | 188 | 304 |
| O-P | 2836 | 158 | 75.5 | 332 |
| I-P | 4974 | 174 | 132 | 364 |
| O-P | 3603 | 177 | 95.9 | 318 |
| I-P | 3982 | 162 | 106 | 339 |
| O-P | 4406 | 221 | 117 | 464 |
| I-P | 4192 | 126 | 111 | 273 |

The data can be used to determine maximum piping stresses for the span by using the maximum measured deflections or velocities in the following equations:

a) $s = K' \frac{D}{L^2}$

b) $s = \left(\frac{S}{Y}\right) Y$

c) $s = \left(\frac{S}{V}\right) V$

s = Stress, psi

D = Pipe O.D., inches

L = Pipe length, ft.

y = Maximum deflection, mils

v = Maximum velocity, ips

$\frac{S}{y}$ = Stress/deflection, psi/mil

$\frac{S}{v}$ = Stress/velocity, psi/ips

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2. Wachel, J.C., von Nimitz, W., "Assuring the Reliability of Off-shore Gas Compression Systems," EUR205, European Offshore Petroleum Conference and Exhibition, 1980 Proceedings, Volume 1, pp. 559-570.