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Analytical Techniques for Evaluation of Compressor-Manifold Response

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This paper discusses the types of vibration and stress problem encountered in reciprocating compressor-manifold piping systems and mathematical analysis methods and digital computer programs developed in the research sponsored by the Pipeline and Compressor Research Council of the Southern Gas Association. These techniques can be used to calculate natural frequencies, mode shapes, vibrational amplitudes, and dynamic stresses caused by acoustical and mechanical excitation forces in the system, and they have been used in the design of domestic and foreign compressor installations representing well over 10 million installed horsepower. Comparison of these predictions with experimental field results has shown that accurate estimates of dynamic stress and impending failures can be obtained.

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Analytical Techniques for Evaluation of Compressor-Manifold Response

J. C. WACHEL F. R. SZENASI

Excessive vibrations and stresses in the piping of reciprocating compressor units are a problem in the natural gas and petrochemical industry. Compressor cylinders and their manifolding bottles form a complex mechanical system subjected to dynamic pulsation forces in the bottles and cylinders. Southwest Research Institute has developed analytical techniques for solving these vibration and stress problems. This research was sponsored by the Pipeline and Compressor Research Council (PCRC) of the Southern Gas Association.

The complete compressor-manifold system is simulated mathematically by defining the stiffness and mass contribution of all structural members. Natural frequencies and mode shapes, both parallel and perpendicular to the engine crankshaft, are calculated. Vibrations and resultant stresses introduced into the nozzles can also be determined. Recent work on the flexibility at nozzle-bottle Junctions has significantly improved the accuracy of the analytical calculations. Compressor installations representing well over 10 million installed horsepower have been designed utilizing these techniques. This paper will describe the basis of the analysis and how it is used in industry to design compressor systems.

COMPRESSOR-MANIFOLD SYSTEM

A typical compressor-manifold system shown in Fig.1 is a gas engine which is driving four compressor cylinders in the horizontal plane. The gas enters the reciprocating compressor cylinders through the suction manifold bottle; it is then compressed and discharged into a discharge manifold bottle. These suction and discharge manifold bottles are usually designed to reduce the pulsations introduced into the gases by the reciprocating compression process. The nozzles which attach the manifold bottles to the compressor cylinders are usually designed to minimize the loading on the cylinders and provide proper acoustical filtering, with considerations also given to their flexibility and pressure drop characteristics.

The primary stress problem in compressor-manifold installations is the nozzle since it

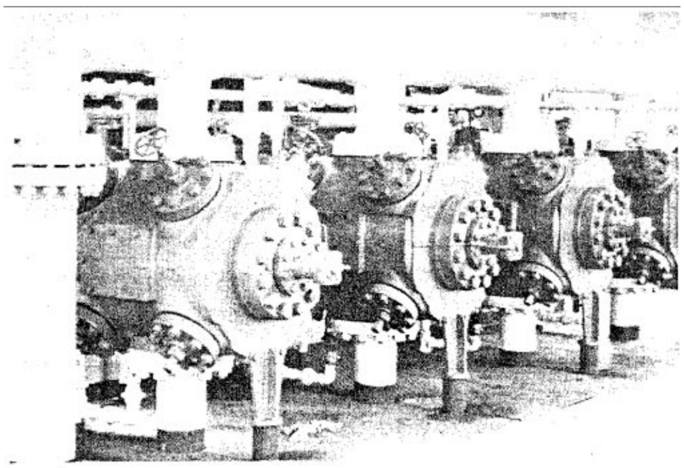

Fig.1 Typical compressor-manifold system

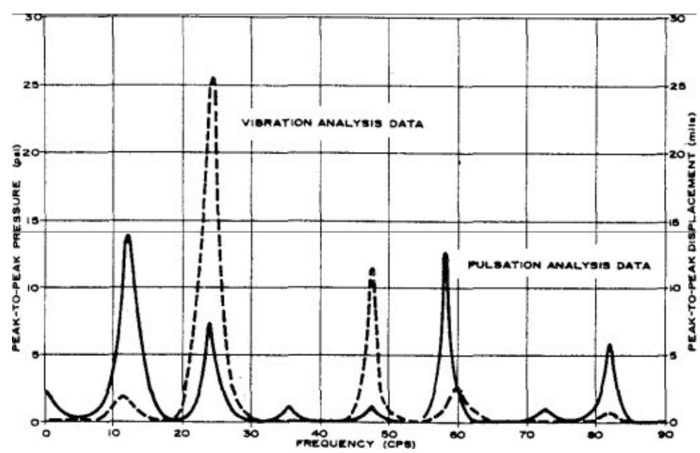

Fig.2 Comparison of measured vibration and pulsations

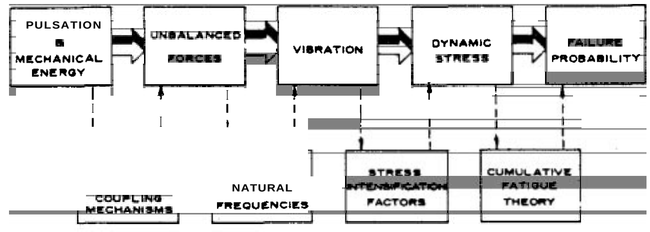

Fig.3 Flow chart for analysis of system reliability

connects the major masses of the system i.e., the suction bottle, discharge bottle, and compressor cylinders. When a vibration resonance of one of these masses occurs, relative deflections between the masses cause dynamic stresses in the nozzles. The resultant stress in the nozzles is a combination of the flexural bending and torsional stresses.

Experience based upon approximately 400 field evaluation studies at reciprocating compressor systems has shown that, whenever failures occur, the cause can usually be traced to the excitation of a mechanical resonance. Reciprocating piston motion generates pulsation energy at every engine harmonic; however, acoustical resonances due to the combination of manifold volumes, nozzles, chokes, internal passage geometry, and the station piping can cause certain frequency components to be amplified. Quite often the acoustical resonance will coincide with a mechanical natural frequency of the system. This resonance condition usually causes excessive vibrations. However, high amplitude pulsations away from a mechanical natural frequency may not cause excessive vibrations. These facts are borne out by the data given in Fig.2.

When a mechanical natural frequency is excited by acoustical pulsations resulting in ex-

NOMENCLATURE

A_{CHG} = cross-sectional area of crosshead guide

A_{DP} = cross-sectional area of distance piece

A_i = constants

BJ_1 = bolted joint inertia of crosshead guide

BJ_2 = bolted joint inertia of distance piece

D = diameter of bottle

d = diameter of nozzle

E = modulus of elasticity

F_{CHG} = shear form factor of crosshead guide

F_{DP} = shear form factor of distance piece

G = modulus of rigidity

I_{CHG} = moment of inertia of crosshead guide

I_{DP} = moment of inertia of distance piece

K_i = generalized stiffnesses

K_{ij} = stiffness matrix

K^{-1}_{ij} = flexibility matrix

L = length of nozzle

L_{CHG} = length of crosshead guide

L_{DP} = length of distance piece

M = generalized moments

M_i = generalized masses

P, F_j = generalized forces

x = deflection parallel to x axis

x_i = generalized displacements

y = deflection parallel to y axis

z = deflection parallel to z axis

β = joint flexibility function

$\delta_{\xi\xi}$ = Kronecker delta

ξ = damping ratio

λ = eigenvalues of stiff'ness-mass matrix

ψ = function

SUBSCRIPTS

C = cylinder

CHG = crosshead guide

DB = discharge bottle

DN = discharge nozzle

DP = distance piece

SB = suction bottle

SN = suction nozzle

$\eta\xi\zeta$ = dummy indices

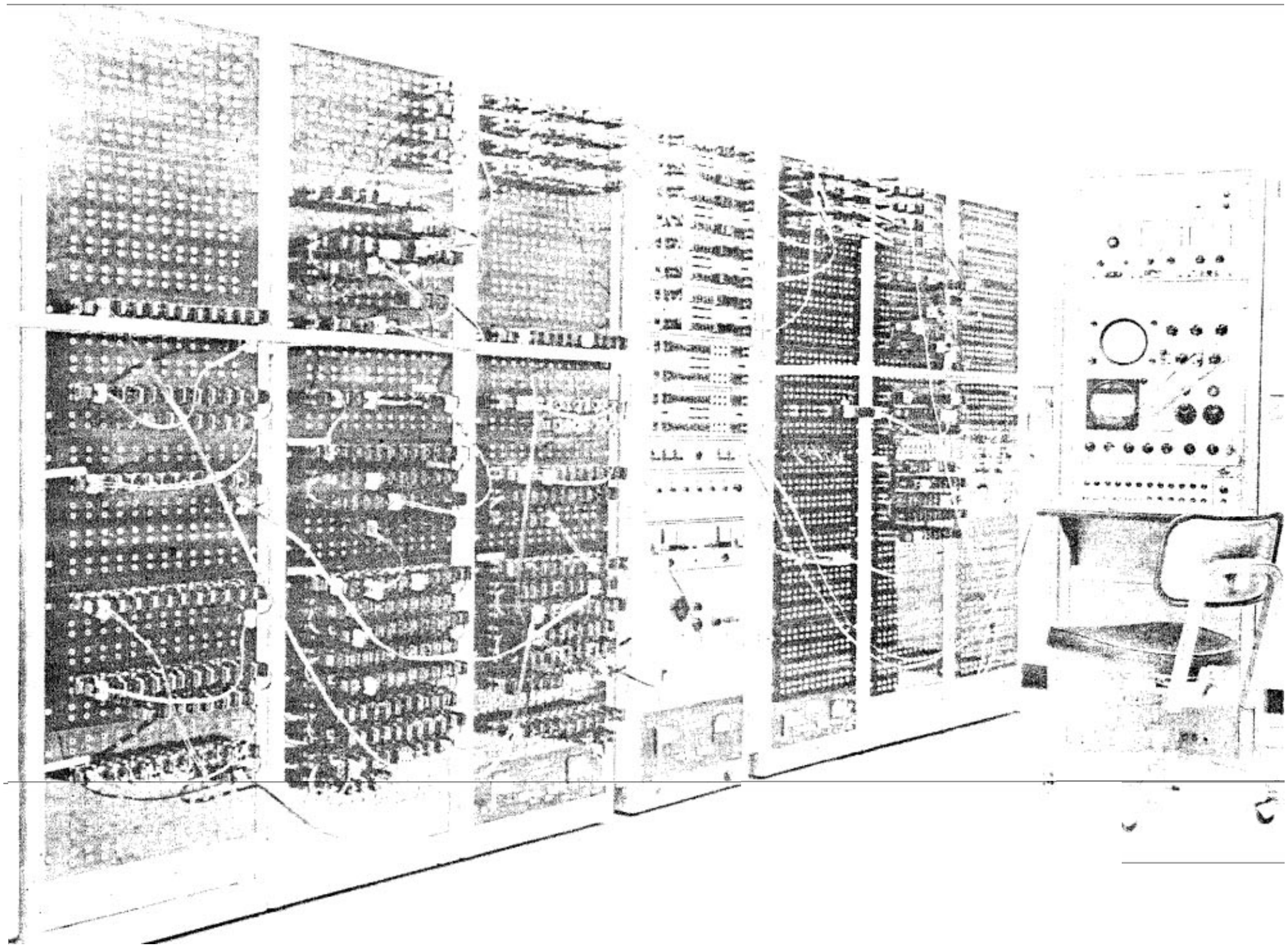

Fig.4 SGA compressor installation design facility

cessive vibrations and stresses, alteration of the design is necessary to eliminate the problem. This solution can be accomplished in either of two ways. One is to reduce the magnitude of the pulsations and unbalanced forces sufficiently to lower the vibration amplitudes to a safe level. The second method is to change the mechanical natural frequency so that coincidence with the major acoustic frequencies does not occur. This will result in a reduction of the vibration amplitudes proportional to the mechanical amplification factor (Q). Changing the nozzle size, length, wall thickness, or the masses of the manifold bottles to change the mechanical natural frequency of the system may also affect the acoustic response. Whenever changes are made in the system, the effects of the acoustic response must be re-evaluated to be sure that no new coincidence of the acoustic and mechanical frequencies occur. As discussed in a previous paper (1),[1] it is necessary to make a complete system analysis in order to insure the adequacy of the system reliability. Fig.3 gives the flow chart for analysis of the system reliability. The ultimate concern of the engineer is whether or not the system will fail. In order to determine this failure probability or the safety factor of the installation, it is necessary to define the pulsation and mechanical energy in the system, the natural frequencies, the unbalanced forces, the vibration amplitudes, and the resultant dynamic stresses. These areas of concern have been emphasized by the Southern Gas Association's PCRC research over the past 15 years. The Southern Gas Association Compressor Installation Design Facility has become the standard design technique for determining the acoustical energy in piping systems. This facility, shown in Fig.4, has been described in several papers (2-5). This system uses an electrical analog model of the acoustic pulsations in the piping system, thereby enabling one to accurately measure pulsations and unbalanced forces in the laboratory. This information can then be used with

[1]Underlined numbers in parentheses designate References at the end of the paper.

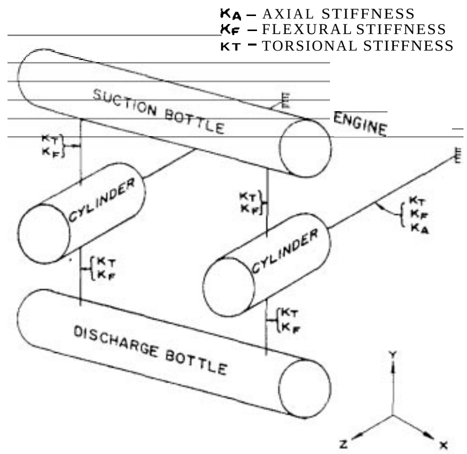

Fig.5 Spring and mass model of compressor-manifold system

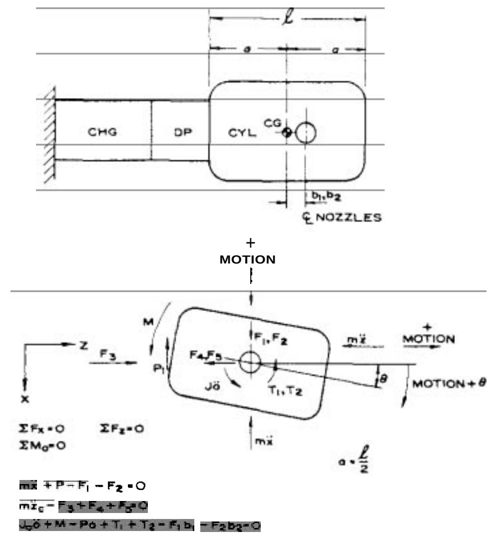

Fig.6 Free-body diagram of cylinder

the mechanical natural frequencies and mode shape data of the compressor-manifold system to predict vibration and stress amplitudes.

CALCULATIONS OF NATURAL FREQUENCIES

A mathematical model of the elastic and mass properites of a compressor-manifold system has been developed which simulates the response of the physical system. The system is conceptually reduced to masses and springs as shown in Fig.5. The nozzles and crosshead guide are represented as flexural, axial, and torsional springs while the cylinders and bottles are masses. The degrees of freedom of the bottles and cylinders are illustrated in Figs.6, 7, and 8. The natural frequencies and mode shapes of the system can be obtained by solving the differential equations of motion for the masses of the system.

The equations of motion which were written for the system are of the form:

$$M_i \ddot{x}_i + C_i \dot{x}_i + K_i x_i = 0 \qquad (1)$$

where:

M_i = i^{th} vibrating mass

C_i = damping on i^{th} mass

K_i = i^{th} spring constant

For simplicity in solving for the natural frequencies the damping terms are allowed to vanish. The percent critical damping in compressor-manifold systems has been determined by experimental field tests to be approximately 0.05 (Q - 10). The difference between the damped and undamped natural frequencies is trivial.

COMPRESSOR CYLINDERS

For the cylinder equations of motion, stiffness of the crosshead guide and distance piece must be defined. The stiff'nesses of the crosshead guide, distance piece, and bolted joints are calculated as springs in series to obtain the effective spring constants for the flexural, rotational and extensional springs (Fig.5). The expression for the combined stiffness of these beams was determined by considering the resulting deflection caused by a unit force acting at the center of gravity of the cylinder (Fig.6). The calculation of deflection considers the shear forces and the slopes at each of the junctions between the different components. The resulting expressions for deflection and slope can be combined into the following equations:

$$x_C = K_{\eta\eta}^{-1} P + K_{\eta\zeta}^{-1} M \qquad (2)$$

$$\theta_C = K_{\zeta\eta}^{-1} P + K_{\zeta\zeta}^{-1} M \qquad (3)$$

where

$$\overline{K_{\eta\eta}^{-1}},\ \overline{K_{\eta\zeta}^{-1}},\ \overline{K_{\zeta\eta}^{-1}},\ \overline{K_{\zeta\zeta}^{-1}}$$

$$= \psi(L_{CHG}, L_{DP}, A_{CHG}, A_{DP}, F_{CHG}, F_{DP}, BJ_1, BJ_2, E, G)$$

The flexibility matrix, $\overline{K_{\eta\zeta}^{-1}}$, which relates the deflections x and θ to the forces P and M is symmetric. In matrix form the equation becomes:

$$\Delta_\eta = K_{\eta\zeta}^{-1} F_5 \qquad (4)$$

where Δ_η represents the displacement vector; $\overline{K_{\eta\zeta}^{-1}}$,

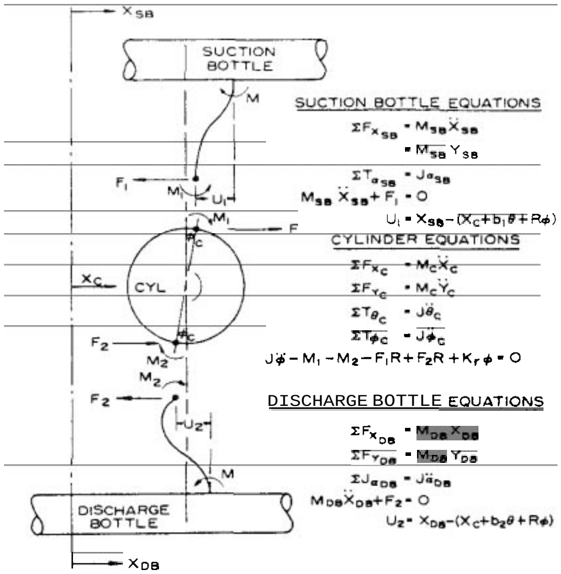

Fig.7 Free-body diagram of system

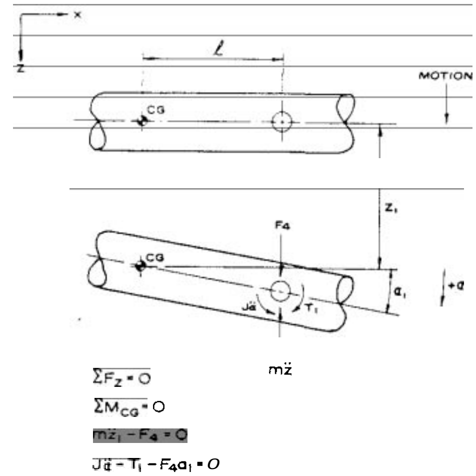

Fig.8 Free-body diagram of bottles

the flexibility matrix; and F_ζ, the force and moment vector.

The calculation of the flexibility matrix inverse which is the stiffness matrix $K_{\eta\zeta}$ allows the solution of the force vector.

$$K_{\xi\eta}\Delta_\eta = K_{\xi\eta}K^{-1}_{\eta\zeta}F_\zeta = \delta_{\xi\zeta}F_\zeta = F_\xi \qquad (5)$$

The forces and moments resulting from deflections of the crosshead guide and distance piece spring are now determined for use in the equation of notion.

MANIFOLD BOTTLES

For the manifold bottles the major springs are the suction and discharge nozzles. The nozzles have forces and moments which cause torsional and bending deflections (Fig.7). An expression for the deflections x_{SB} and $\overline{\phi_C}$ can be written in terms of the forces and moments imposed upon the nozzle from the cylinder similar to equations (2) and (3). For these displacement vectors

$$\overline{K^{-1}_{\eta\eta}}, \overline{K^{-1}_{\eta\zeta}}, \overline{K^{-1}_{\zeta\eta}}, \overline{K^{-1}_{\zeta\zeta}} = \overline{\psi(L_{SN}, I_{SN}, F_{SN}}, D_{SN}, \beta_{SN}, E, G) \qquad (6)$$

The equations for the discharge bottle can be written similarly.

The other degrees of freedom of the bottles are illustrated in Fig.8. The motions are a combination of linear and angular movements from the equilibrium position. The calculation of the forces and torques on the bottles require the determination of the torsional and flexural spring stiffnesses. The forces acting on the bottles as a result of the linear and angular motion can be determined as a function of the stiffnesses. The resulting equations for z_{SB} and α_{SB} can be solved as in equations (2) and (3). For these vectors

$$\overline{K^{-1}_{\eta\eta}}, \overline{K^{-1}_{\eta\zeta}}, \overline{K^{-1}_{\zeta\eta}}, \overline{K^{-1}_{\zeta\zeta}} = \Psi(\overline{L_{SN}}, I_{SN}, A_{SN}, D_{SN}, \beta_{SB}, E, G) \qquad (7)$$

All of the equations of motion are now in the form:

$$M\ddot{X} + KX = 0 \qquad (8)$$

The differential equations of motion were converted into matrix form for simplicity of solution. The general form of the equation is

$$M\ddot{X} = -KX \qquad (9)$$

where M is the diagonalized mass matrix and K is the stiffness matrix. The form of these equations lends itself quite readily to the eigenvalue-eigenvector solution method.

The relationship between x and $\ddot{X}$ ($\ddot{X} = -\omega^2 x$) is used to simplify the equation of motion:

$$-M\omega^2 X = -KX \qquad (10)$$

where the ω^2 represents the diagonalized eigenvalue matrix which is called lambda (λ).

$$M\lambda X = KX \qquad (11)$$

The diagonalized mass matrix inverse is obtained by forming a matrix whose diagonal is the individual inverse of the individual masses. Multiplying both sides of the matrix equation by M inverse (M^{-1}) gives

$$\lambda X = M^{-1}KX \qquad (12)$$

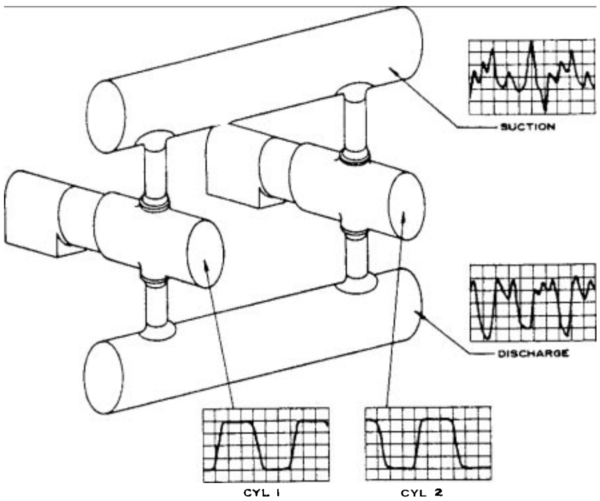

Fig.9 Excitation forces in compressor-manifold system

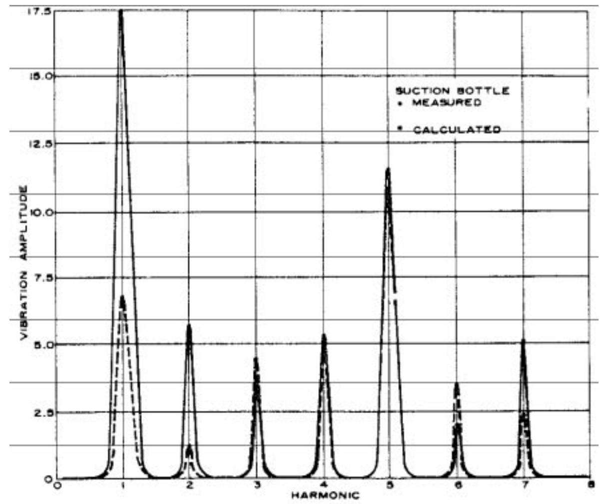

Fig.10 Comparison of calculated and measured suction bottle vibrations

Rearranging gives:

$$(M^{-1}K - \lambda)X = 0 \qquad (13)$$

This form of a matrix equation is the familiar eigenvalue equation. The values of λ for which the equation is soluble are known as the characteristic values or eigenvalues of the matrix. The problem of finding the vectors which satisfy the equation is therefore called the eigenvalue problem for the given matrix. Correspondingly, the vector solutions are the eigenvectors of the matrix $(M^{-1}K)$, which is referred to as the stiffness-mass matrix.

Physically, the eigenvectors represent the mode shapes of the particular vibration corresponding to the eigenvalue which represents the vibrational natural frequency. The equation obtained when the determinant of the coefficient matrix vanishes is known as the characteristic equation of the matrix and the values of λ for which the equation is satisfied are the desired eigenvalues. In general, the characteristic equation will have "n" roots with "n" eigenvectors.

The solution for the eigenvalues of a system with "n" degrees of freedom would be an "n^{th}" order equation whose solution would give "n" roots of the characteristic equation. The "n" roots would then represent the vibrational frequencies squared. The characteristic equation of the example would be as follows:

$$\lambda^n + A_{n-1}\lambda^{n-1} + A_{n-2}\lambda^{n-2} + \cdots A_2\lambda^2 + A_1\lambda^1 + A_0 = 0 \qquad (14)$$

where A_i, $i = 0, n$ represents functions of mass and stiffness.

When this characteristic equation is solved for the roots or eigenvalues, then the eigenvectors can be obtained. The eigenvectors are obtained using equation (12). Using the stiffness-mass matrix $(M^{-1}K)$ and multiplying by an eigenvector X and forcing this to equal an eigenvalue times the unknown eigenvector will result in a set of equations. This set of equations is soluble for a unique eigenvector direction; however. the magnitude remains undetermined. These eigenvectors represent the mode shape of vibration corresponding to the eigenvalue or natural frequency of that vibration.

Table 1 Calculated versus measured natural frequencies

	Mode	$f_{calc.}$	$f_{meas.}$
Unit A	rigid body (x)	25	24
	suction bottle (z)	42	38
	cylinder (x)	58	58
Unit B	rotary (x)	67	73
	cylinder (x)	39	39
	suction bottle (z)	28	29
	discharge bottle (z)	19	20
Unit C	rigid body (x)	18	15
	rotary (x)	76	72
	discharee bottle (z)	33	31

MODES OF VIBRATION

A resonant mode shape normally occurs for each degree of freedom. The lowest vibration mode in a typical system is generally the z re-

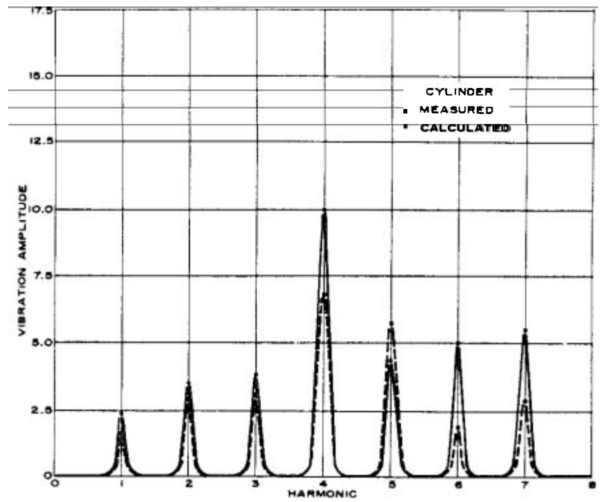

Fig.11 Comparison of calculated and measured cylinder vibrations

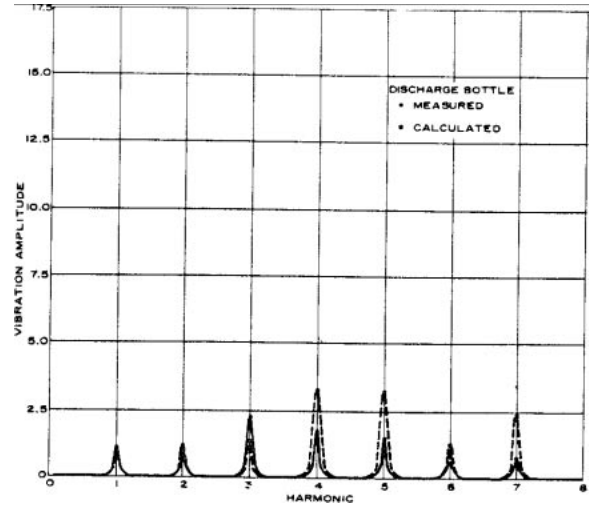

Fig.12 Comparison of calculated and measured discharge bottle vibrations

sonance of the manifold bottles. If the bottle is not completely symmetric with respect to the cylinder and nozzle locations, a combined a-- z resonance of the bottle will occur.

In the x direction which is parallel to the bottle axes, several resonant modes occur which are of interest in practical problems. A low frequency rigid body motion of the manifold bottles and all cylinders is usually the predominant mode shape. Another x resonant mode is the rotary mode in which the two bottles move in opposite x directions and the cylinders remain at rest. In addition a cylinder resonance mode occurs in which the manifold bottles remain at rest while the cylinders are at resonance.

Other resonances which occur at higher frequencies are the cylinder z resonant mode which occurs colinear with the cylinder longitudinal axis. The cylinder ϕ resonance is a mode shape in which the cylinder rotates about its longitudinal axis.

CALCULATION OF VIBRATION AND STRESS

A computer program which uses the eigenvector method for solving the mechanical natural frequencies and mode shapes of the compressor-manifold system enables one to calculate the forced vibration response of the system using various forcing functions at different mass locations. A Fourier expansion of any complex forcing function, including the phasing, can be applied at each mass location. Complex waves of typical unbalanced forces in the suction and discharge bottles and cylinders are shown in Fig.9. These forcing functions can be obtained from experimental field data or the SGA compressor installation design facility.

The harmonics of the complex forcing functions simulating the magnitude and direction of the actual forces encountered in the system are applied to the mathematical model. The dynamic displacements resulting from the applied harmonic forces of all the masses can be calculated. Vibrations at various harmonic frequencies can be recombined as a complex vibration waveform. Stress

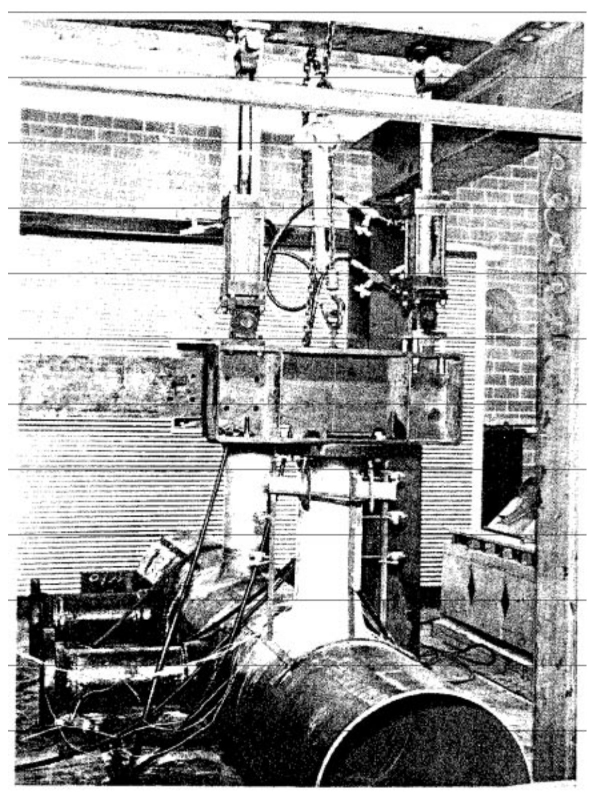
Fig.13 Experimental facility for branch connection research

values based upon the relative deflections between connecting members are calculated for each harmonic frequency. Flexural bending and torsional stresses in the nozzles calculated for each type of motion are then combined into the resultant maximum shearing and principal stresses by the combined stress equations.

CORRELATION WITH FIELD STUDIES

Mechanical natural frequencies and mode shapes measured during field evaluation studies of the compressor-manifold system have correlated well with calculated results. A comparison of measured and calculated natural frequencies of the most commonly occurring modes is given in Table 1.

In addition, vibration deflections and dynamic stresses are routinely measured on field evaluation studies and are compared with the corresponding vibrations and stresses calculated by the previously described analytical techniques. A comparison of the calculated and measured values of vibrations for a typical field evaluation study are presented in Figs.10, 11, and 12. While correlation of individual data points varies, the overall trends compare favorably.

When field data on the vibrational amplitudes of the compressor-manifold system are available, this analytical technique can be used to predict the stresses occurring in the system. The measured vibrational amplitude at a point can be used to determine the vibrational amplitudes and stresses of the system at a particular frequency.

FLEXIBILITY IN NOZZLE-BOTTLE JUNCTIONS

To accurately calculate natural frequencies and stresses it is necessary to consider the nozzle-bottle joint flexibility factor. The short, stiff nozzles typically used in compressor-manifold systems cause the bottle wall to deflect and the joint to rotate. Analysis of these deflections is complicated by the use of reinforced branch connections such as saddles, pads, sweepolets, weldolets, drawn outlets, encirclement saddles, and tees. Experimental data for each reinforced branch connection are necessary to determine the joint flexibility function, since the complex geometry is not readily solvable by simple shell theory. Data on several types of branch connection designs has been obtained by others (6).

The authors have conducted static and dynamic tests on several nozzle-bottle reinforced branch connections including saddles, sweepolets, pads, and weldolets. These tests determined the flexibilities and stress intensification factors for bending, shear, and torsional loads. In addition the effect of the various reinforcements upon the mechanical natural frequencies of the system were obtained. The experimental facility used to make these measurements is shown in Fig.13.

Equations which can be used to calculate the joint flexibility functions have been obtained from analysis of the measured data.

CONCLUSIONS

This paper has described methods which are part of a continuing effort by the Southern Gas Association ana Southwest Research Institute to predict the response of compressor-manifold systems. Some of the major conclusions which can be made are:

1 Comparison between calculated and measured mechanical natural frequencies has shown agreement within accuracy requirements needed for design of reciprocating compressor installations.

2 vibrations of the compressor-manifold system can be calculated by using the acoustical forces in the mathematical model. Measured and predicted vibration amplitudes compare favorably.

3 The joint rotation at the nozzle-bottle connection greatly affects the calculation of natural frequencies, mode shapes, vibration, and stresses. Research conducted during the past year has resulted in improved equations for prediction of this flexibility, and methods for the selection of the optimum fitting for particular loading conditions have been evolved.

4 The analytical techniques described can be used in the design stage in conjunction with the Southern Gas Association compressor installation design facility to study the combined effects of acoustical and mechanical changes. The dynamic stresses and fatigue life of the compressor nozzles can be predicted.

5 In existing installations it is possible to estimate the stresses in the compressor nozzles caused by vibrations measured on the actual installation by forcing the mathematical system to have the same vibration amplitudes. By using this technique an incipient failure can be corrected before it occurs.

REFERENCES

1 Wachel, J.C., "Consideration of Mechanical System Dynamics in Plant Design," ASME Paper No. 67-DGP-5.

2 Nimitz, Walter, and Damewood. Glenn, "Com-

pressor Installation Design Utilizing an Electro-Acoustical System Analog," ASME Paper No. 61-WA-290.

3 Henderson, E.N. , "Gas Pulsations—The Problem, Southern Gas Association's Approach, Results," Oil & Gas Journal, Vol. 56, No. 19, 1958.

4 Hughes, J.V., and Sharp, J.M., "The Design and Operation of an Electrical Analog of a Reciprocating Compressor Installation," ASME Paper No. 56-A-200, 1956.

5 Damewood, Glenn, "SGA and Southwest Research Team Up to Take the Pulse of Compressor piping," Pipeline Industry, July, 1956, pp. 30-33.

6 Rodabaugh, E.C., et al., "Evaluation of Experimental and Theorectical Data on Radial Nozzles in Pressure Vessels," Section VI, USAEC, TID-24342.