

VIBRATION ANALYSIS OF VERTICAL PUMPS

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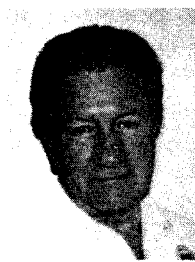
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ABSTRACT

Vibration field data was measured on several large motor-driven vertical cooling water pumps which experienced excessive wear of the impellers, wear rings and seals after a short period of operation. The data indicated that the problem was due to the operating speed being near the pump-motor system

mechanical natural frequency, which resulted in excessive vibration levels on the motors and pump impellers. The mechanical natural frequency was very sensitive to the effective stiffness of the connections between the concrete, baseplates, pump base and motor flange. Tests were conducted to evaluate the advantages and disadvantages of lowering the natural frequency below the running speed and raising the natural frequency above the running speed. The results of these tests are presented with conclusions and recommendations.

INTRODUCTION

In October 1984, Tenneco Oil Company installed six vertical single stage pumps to provide cooling water to their refinery (Figure 1). There are four electric motor-driven pumps and two emergency diesel engine-driven pumps. The pump design data and operating conditions follow.

| | |
|---------------------------|------------|
| Design Flow | 20000 gpm |
| Minimum Intermittent Flow | 4000 gpm |
| Suction Pressure | - 10 psig |
| Differential Pressure | 75 psig |
| TDH | 174 feet |
| Submergence | (4 ft req) |
| Speed | 885 cpm |
| Horsepower Required | 987 Bhp |

After operating less than two months, all of the electric motor-driven pumps experienced seal failures, bearing failures, and excessive wear on the impellers and wear rings. The seal failures allowed water to contaminate the bearing grease which contributed to the bearing failures. When the seals were worn, the water forced the grease out of the top seal near the coupling, between the motor and pump. The impeller and shaft were worn primarily on one side, while the stationary parts indicated excessive wear completely around the circumference.

Mechanical Checks

Visual observation and vibration measurements at the top of the motor indicated that the motor vibration levels were excessive. The vibration levels at the top of the motor were often above 20 mils peak-to-peak which exceeds the Hydraulic Institute acceptable vibration limit of six mils for vertical pumps at 900 cpm [1]. Although the vibration levels were excessive, it was not known whether the motor vibrations were related to the wear problems on the pump. A comprehensive program was

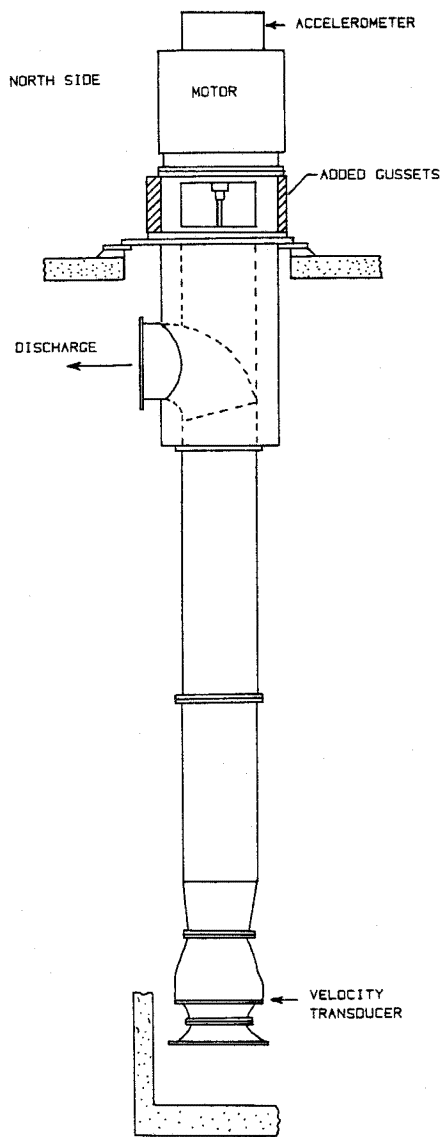


Figure 1. Vertical Pump Assembly.

then developed to determine the mechanical integrity of the individual pumps. The following mechanical tests were done:

- Several of the motors were run solo and vibration readings were made on the motors to compare with the previous coupled data.
- Efficiency checks were made on each pump.
- Each coupling was checked for fit, concentricity, and dynamic balance.
- The pump mounting heads were checked for parallel machining and level installation.
- The motor alignment to the pump was verified.
- The impellers were dynamically balanced.
- The pumps shafts were checked for straightness.
- The pump bases were removed and re-installed with proper shims.
- The pump internal clearances were checked.
- Additional lubrication points were installed to lubricate the pump shaft.

Lubrication Problem

These tests revealed that the pumps were generally installed correctly, and that the dimensions and tolerances were as specified by the pump and motor manufacturers. However, the tests did identify that the pumps had a basic lubrication problem. It was found that the grease system would not develop enough pressure to overcome the system back pressure. This resulted in frequent lubrication trip-outs, and the pumps would not receive proper lubrication until an operator checked the pumps. Inspection of the bearings revealed that the grease could not pass through the bearings due to the lack of grease passages. The lubrication problems were corrected by installing grease pumps which maintained a constant flow of grease into the pump, by redesigning the bearings with longitudinal grease passages, and by adding a third grease point just above the fourth bearing from the top.

Although the modifications improved the pump lubrication, the seal and impeller failures were not eliminated. This suggested that the problem was a design problem rather than an installation or maintenance problem.

Sump Vortex

Initially the pump manufacturer suspected that the pump vibration could be the result of vortices in the sump. Visual observation of the water surfaces in the sump showed possible vortex circulation at several locations. Vortices at pump intakes are a well known source of problems including: 1) reduced pump efficiency, 2) vibration and noise, 3) increased wear of bearings, and 4) accelerated deterioration of impeller blades due to abrasion by entrained debris, corrosion due to excessive air, and pitting [2].

A review of the pump design indicated that the design generally conformed with the Hydraulic Institute guidelines [1]. Although the sump design appeared satisfactory, the pump manufacturer felt that a more detailed analysis of the sump should be made. However, since the time to model a sump can typically require several months to complete, the manufacturer installed a vortex suppressor (vortex splitter in the inlet strainer) in an effort to eliminate a possible vortex at the pump inlet. The detailed sump analysis was not performed.

Vibration data recorded on the pump with the vortex suppressor indicated that the vibration levels were not reduced. While there may have been evidence of vortices in the sump, it was concluded that the excessive pump vibration was not caused by vortices at the pump inlet.

Vibration Transmission

The pumps were mounted on a common concrete structure and discharged into a common header which provided transmission paths for the vibration between the pumps. It was observed that the motor vibration amplitudes would significantly increase and decrease every few minutes (beating phenomenon) as the vibration of the various pumps were in-phase and out-of-phase.

In an effort to reduce the vibration transmission between the pumps and the discharge piping, a rubber bellows was installed in the discharge piping of one of the pumps. The vibration levels on the pump were not reduced which meant that isolating the discharge piping would not solve the vibration problem.

FIELD EVALUATION

The user company requested that the consulting engineers assist in determining the causes of the failures and to recommend modifications to solve the problems.

Instrumentation

In recent years, it has become fairly common practice to install vibration monitoring systems on large equipment. How-

ever, vertical pumps, such as the cooling water pumps, are generally not equipped with vibration transducers. It is difficult and expensive to install underwater instrumentation to measure vibrations near the impeller. Generally, the vibrations are monitored near the top of the motor, because it has been found that the motor vibration can sometimes be indicative of the vibration on the impeller [3].

To obtain vibration data on the pump column, the pump manufacturer installed two velocity transducers near the impeller to measure the vibration in the East-West (E-W) direction, perpendicular to the discharge flow, and in the North-South (N-S) direction, parallel to the flow (Figure 1). The transducers were mounted on a pump before it was reinstalled in the sump. In underwater service, the velocity probes have advantages, compared to other transducers, such as accelerometers and proximity probes, because the velocity transducers generate an electrical signal without any power supply or signal conditioner. The probes operated successfully in the river water for several months without any special waterproofing.

Ideally, vibration data should be taken at several locations between the impeller and the top of the motor, to accurately define the vibration mode shape. However, these test points were inaccessible, and for safety reasons no personnel could be in the sump when the pumps were in operation. Therefore, the vibration measurements were limited to areas above the top of the deck and the velocity transducers on one pump.

Vibration Versus Discharge Pressure

Operation with several pumps in service raised the system discharge pressure and increased the vibration levels and beating between the pumps. Therefore, tests were done to determine if the increased vibration levels were directly related to the increased discharge pressure. Due to operational constraints, the system discharge flowrates and pressure could not be tested over a wide range. Consequently, it was decided to increase the pressure on individual pumps by throttling the block valve at the pump discharge.

The flowrates were estimated using the pump performance curves and the motor horsepower. The motor horsepower was calculated from the measured motor amperage.

The tests were made on two pumps: G-8405, which was recently overhauled and thought to be in good mechanical condition, and G-8403, which had the velocity probes installed near the impeller. As shown in Table 1, the vibration amplitudes remained fairly constant for the range of discharge pressures tested.

The measured total head vs horsepower curves generally agreed with the curve supplied by the manufacturer. Pump G-

8405 was in better mechanical condition and the performance was closer to the design curve compared to G-8403, which had higher vibration levels and increased seal clearances. This test indicated that the pumps were operating near the design flow condition, and that the vibration levels were not significantly affected by the discharge pressure.

Structural Mechanical Natural Frequency

A variable speed mechanical shaker attached to the top of the pump motor was used to excite the motor-pump mechanical natural frequencies (Figure 2). The shaker speed was varied from approximately 400 cpm to 1200 cpm, to identify the mechanical natural frequencies. The vibration levels at the shaker running speed and the vibration phase angle relative to the shaker key-phaser signal were plotted versus shaker speed (Bodé plot). The data were also plotted in a polar format (Nyquist plot) with the amplitudes plotted 1/4 scale.



Figure 2. Variable Speed Shaker Mounted on the Motor.

The mechanical natural frequencies were measured on several units, but most of the testing was done on pump G-8403. As shown in Figures 3 and 4, the mechanical natural frequency in the E-W direction was 870 cpm on the top of the motor and near the impeller. This test illustrated that the response at the top of the motor was proportional to the vibration at the impeller and was not a response of the motor alone.

Attachment Stiffness

The testing showed that the mechanical natural frequency was a direct function of the stiffness of the bolted connections between the concrete, the pump, and the motor. To illustrate this effect, the anchor bolts were loosened and the natural frequency was reduced from 870 cpm to 840 cpm (Figure 5). The vibration amplitude at the pump running speed was reduced

Table 1. Pump Vibration vs Discharge Pressure.

| Pump G-8405 | | | | | | |
|------------------------|----------------------|------------|------|-------------|-------------|--|
| Disch press psig | Total Head ft. | Motor Load | | Motor Vib | | |
| | | Amps | HP | E-W mils | N-S mils | |
| 60 | 147.4 | 127 | 1021 | 7-10 | 4-5 | |
| 65 | 159.0 | 125 | 1005 | 7-9 | 5 | |
| 70 | 170.5 | 122 | 981 | 6-7 | 2-3 | |
| 75 | 182.0 | 117 | 940 | 6-7 | 3-4 | |
| 80 | 193.6 | 108 | 868 | 7-9 | 5-6 | |

| Pump G-8403 | | | | | | | |
|------------------------|----------------------|------------|-----|-------------|-------------|---------------|---------------|
| Disch press psig | Total Head ft. | Motor Load | | Motor Vib | | Impeller Vib | |
| | | Amps | HP | E-W mils | N-S mils | E-W in/sec | N-S in/sec |
| 60 | 147.4 | 123.5 | 993 | 11.4 | 7.3 | 0.24 | 0.26 |
| 65 | 159.0 | 121.0 | 973 | 11.8 | 5.5 | 0.26 | 0.20 |
| 70 | 170.5 | 117.0 | 940 | 14-15 | 3-5 | 0.30 | 0.17 |
| 75 | 182.0 | 109.0 | 876 | 13.5 | 2-5 | 0.32 | 0.17-.2 |
| 80 | 193.6 | 104.0 | 836 | 12.5 | 2-5 | 0.28-.3 | 0.14-.23 |

Total Head = 2.307 * (psig) + 9 ft. (distance of gage to water)

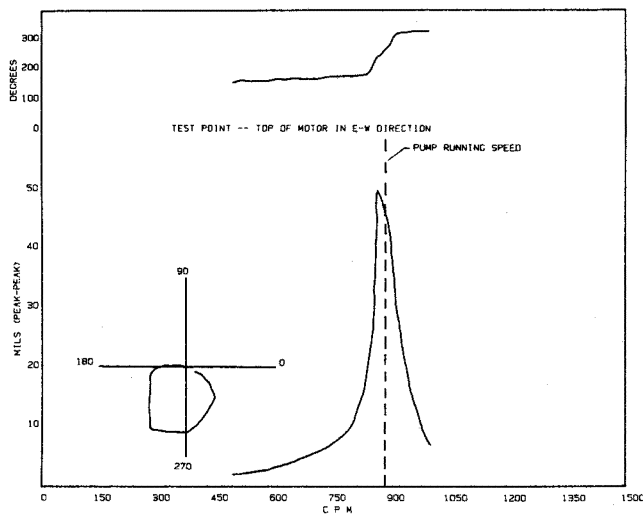


Figure 3. Natural Frequency of Original System—Measured at the Top of the Motor.

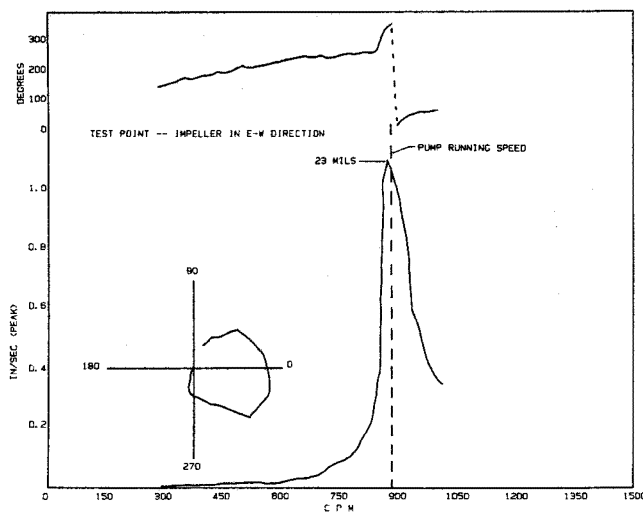


Figure 4. Natural Frequency of Original System—Measured at the Impeller.

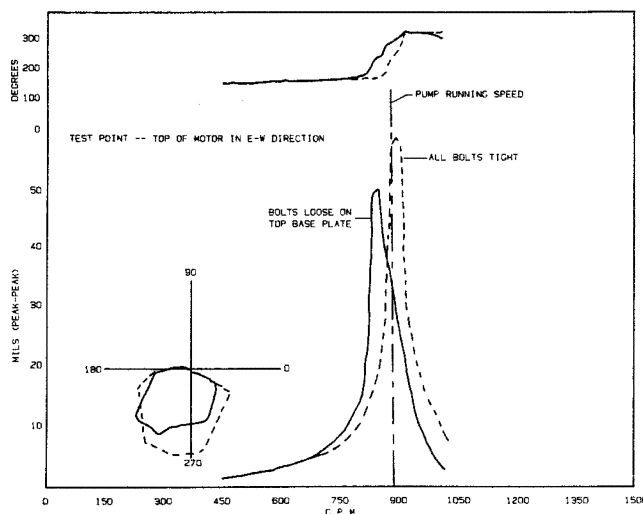


Figure 5. Effect of Anchor Bolt Torque on Natural Frequency.

from 45 mils to 35 mils, because the response was further removed from the pump running speed.

All of the bolts were then retightened, and the resonance was increased from 840 cpm to 900 cpm (Figure 5). The motor vibration amplitude at the pump running speed was increased to 60 mils. These tests indicated that the vibration amplitudes at the pump operating speed were actually lower with the anchor bolts loose, because the natural frequency was reduced to 840 cpm.

The pump base plate was rectangular, which made the anchor bolts further from the pump in the N-S direction. This increased distance to the anchor bolts made the base more flexible in the N-S direction, compared to the E-W direction. The increased flexibility lowered the mechanical natural frequency in the N-S direction further below the pump running speed, which explains why the vibration amplitudes were generally lower in the N-S direction.

These tests confirmed that the response of the baseplate was very sensitive to the effective stiffness of the connection to the concrete. The effective attachment stiffness varied considerably between each of the pumps, due to the tightness of the anchor bolts and the alignment shims. This explained the difference in the mechanical natural frequencies between the "identical pumps" and why some of the pumps were more vibration sensitive.

Temporary Braces

The field data indicated that the pumps were vibrating individually and were not necessarily phase related to the adjacent units. One method often used to increase the stiffness of a system is to physically tie the individual units together. To evaluate this effect, a portable hydraulic jack was used to wedge a 4 in \times 4 in wooden timber between the tops of two adjacent motors.

The timber reduced the vibration amplitudes from 50 mils to 25 mils at the top of the motor and from 1.1 ips (23 mils) to 0.64 ips (13.8 mils) at the impeller, as shown in Figures 6 and 7. The vibration reduction appeared to be primarily the result of increasing the damping in the system rather than stiffening the system because the natural frequency was not significantly changed.

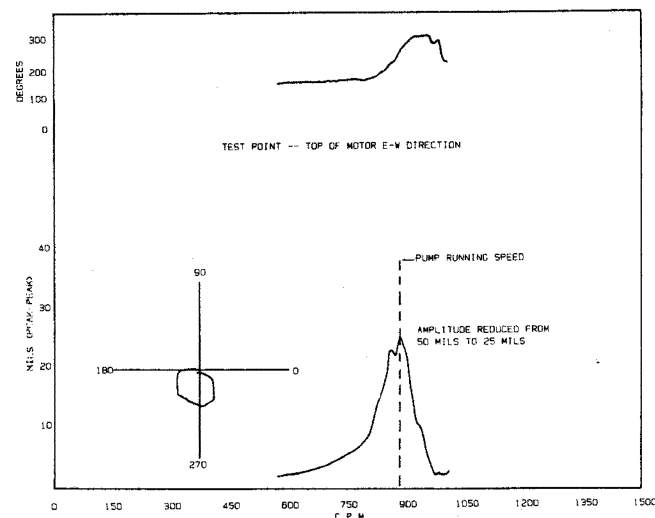


Figure 6. Effect of Temporary Brace—Measured at the Top of Motor.

This test indicated that it would be possible to reduce the motor and impeller vibration levels by tying the units together.

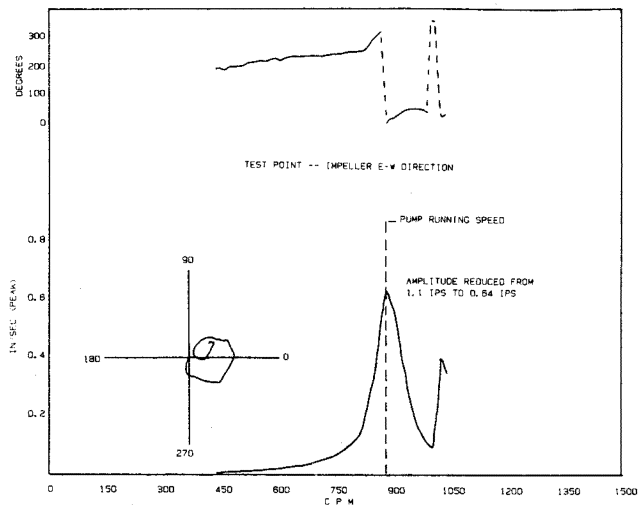


Figure 7. Effect of Temporary Brace—Measured at the Impeller.

While the braces would have provided a temporary solution, they were not recommended because: 1) it would have been difficult to attach the braces to the motor, 2) the pumps on the ends were adjacent to the shorter diesel-driven units and the end pumps could be braced only on one side, and 3) the motor manufacturer would not approve the braces as a long term modification.

Modification to Move the Mechanical Natural Frequency

The tests demonstrated that the vibration levels could be reduced by moving the mechanical natural frequency further from the running speed. The natural frequency could be lowered by mounting the pump on a soft base, or the frequency could be raised above the running speed by rigidly attaching the upper baseplate to the lower baseplate and stiffening the motor mounting base.

Soft Base. Neoprene isolator pads were installed between the upper and lower base plates to lower the pump-motor system mechanical natural frequency further below the pump running speed. The anchor bolts, which attached the upper and lower base plates, were isolated from the bases with rubber sleeves and rubber washers to ensure that there was no direct mechanical coupling between the bases. The bolts were "finger tight" to avoid crushing the pad.

The shaker tests revealed that the system mechanical natural frequency was lowered to approximately 660 cpm in the N-S direction and 720 cpm in the E-W direction (Figures 8 and 9). The vibration amplitudes at the pump running speed were reduced by a factor of approximately six-to-one on both the motor and the impeller.

Vibration data recorded with all the motor driven pumps running showed that the motor vibration at the pump running speed was reduced from 13.7 mils to 2 mils in the N-S direction and from 3.3 mils to 2.8 mils in the E-W direction. Although the vibration amplitudes were generally reduced at the pump running speed, the beating between the other pumps was still present. Also, the soft mounting made the system more sensitive to random low level turbulence which caused increased vibration at the system natural frequencies.

Rigid Base. Additional bolts were added to rigidly attach the upper base to the lower base plate. Gusset plates were temporarily installed to stiffen the motor base (Figure 10).

The system natural frequencies were increased to 950 cpm in the N-S direction and 975 cpm in the E-W direction (Figures 8 and 9). While the natural frequency was still within nine percent

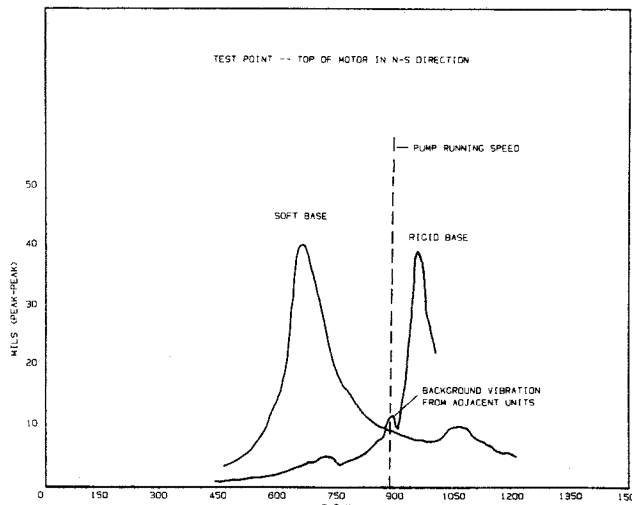


Figure 8. Comparison of Motor Response in the N-S direction with Soft and Rigid Bases.

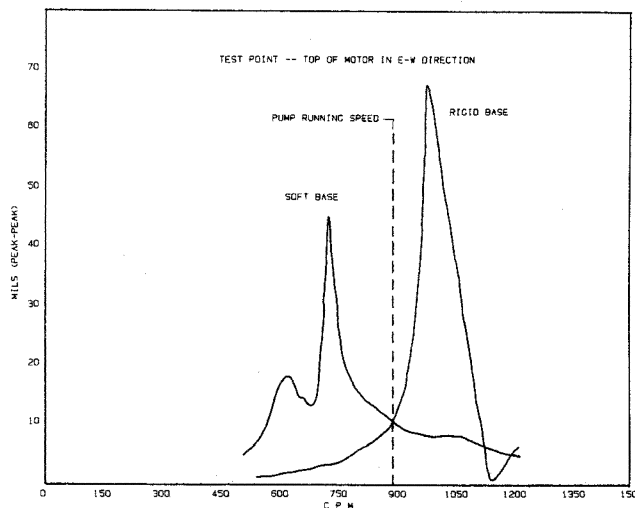


Figure 9. Comparison of Motor Response in the E-W direction with Soft and Rigid Bases.

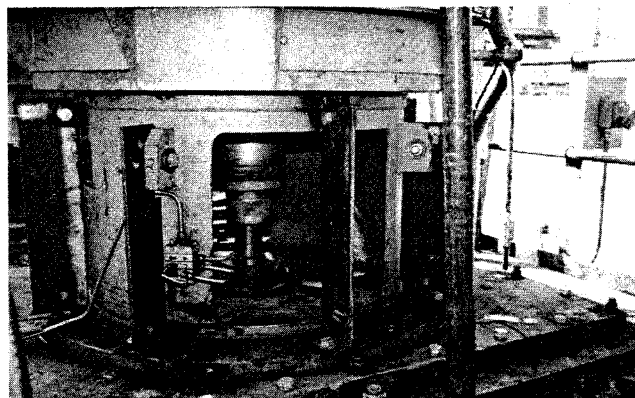


Figure 10. Gusset Plates and Additional Anchor Bolts.

of the running speed, the amplitudes at the pump running speed were reduced approximately four-to-one.

Data recorded with the pumps in operation indicated that the vibrations were primarily at the pump running speed. The vibration amplitudes were reduced by a factor of 5.0 in the E-W

direction and a factor of 2.5 in the N-S direction, compared to the data before the modification. The amplitude reduction favorably agreed with the shaker data which indicated a four-to-one reduction.

DISCUSSION

Hard vs Soft Mounting

The field tests indicated that the neoprene pads were effective in lowering the system natural frequency which reduced the running speed vibration. The data also illustrated that the additional anchor bolts and gusset plates increased the natural frequency and reduced the running speed vibration. Each technique has advantages and disadvantages which should be considered when determining the optimum solution to reduce the pump vibration and resulting wear.

Soft System (Neoprene Isolation Pad)

Advantages. The major advantage of the isolation pad was that it was fairly simple to install, and required no structural modifications. The natural frequency was lowered approximately 180 cpm (20 percent below the running speed) which reduced the running speed vibration. Also, the pad increased the isolation from structure-borne vibration.

Disadvantages. The system had low stiffness, and the vibration produced by random turbulence caused the vibration levels at the system natural frequencies to be a significant portion of the overall total vibration. Also, the vibration at the running speed due to unbalance could be increased because the system was more compliant (less stiff). A potential problem was that the neoprene could deteriorate and lose its elastic properties, thus allowing the natural frequency to move closer to the running speed. Another disadvantage was that it required a large crane to lift the entire pump assembly for installation of the pads.

Stiff System (Gussets and Anchor Bolts)

Advantages. The stiff system offered several advantages in comparison to the soft system. The system natural frequency was effectively increased above the running speed, which reduced the vibration at the running speed. Increasing the natural frequency above the running speed also prevented the natural frequency from being excited during startups and shutdowns. Another advantage was that the system was stiffer and less sensitive to low level turbulence and increases in unbalance. Of importance from a maintenance point of view was that the extra anchor bolts and gussets were installed on the existing pumps without removing the pumps from their bases.

Disadvantages. The gusset plates and anchor bolts were costly to install, in comparison to the isolation pads. The company chose to temporarily attach the gussets with bolts, to avoid a potential alignment problem. Also, the anchor bolts were drilled and tapped, which increased the costs and installation time. The bolting created potential problems, because the natural frequency could be lowered closer to running speed if the bolts loosened. This potential problem could be avoided, however, by welding the gussets and the baseplates instead of using the bolts.

The company conducted additional tests to accurately evaluate the vibration characteristics with the soft and hard mountings. The data indicated that while the soft mounting reduced the overall amplitudes compared to the original design, the vibration levels with the rigid mounting were even further reduced (Figure 11).

Therefore, the preferred solution was to stiffen the base. All of the pump bases were similarly stiffened, and the pump vibration levels were reduced and, more importantly, the wear

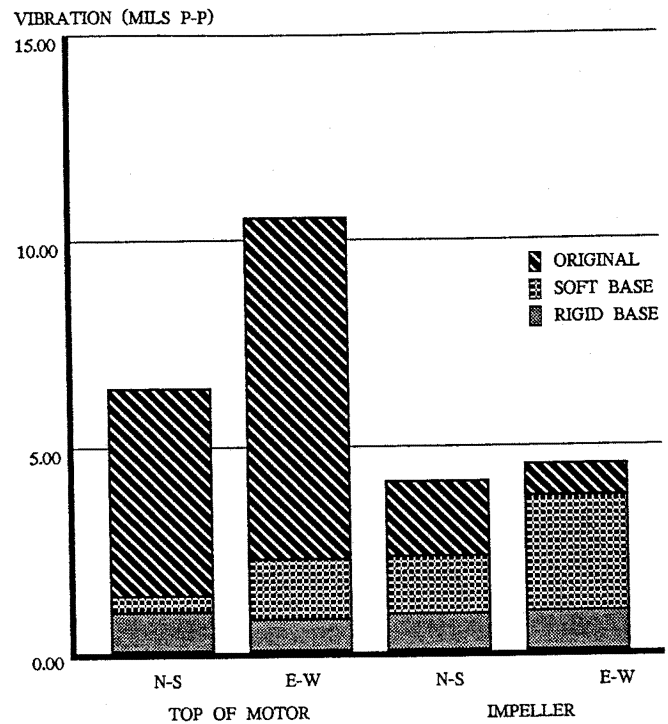


Figure 11. Comparison of Overall Vibration Levels with the Different Bases.

failures were eliminated. The pumps have been in service for approximately one year with the stiffened bases, and there have been no reported failures from the excessive vibration.

System Natural Frequency

The term "reed critical frequency" is often used in the pump industry when referring to the mechanical natural frequency of vertical pumps and motors [4]. In the past, many people have analyzed the motor-pump system as two different independent systems with the motor and pump individually attached to a rigid mass (Figure 12).

The motor manufacturer quoted the motor reed critical frequency to be 1650 cpm for the motor bolted to a rigid support and considered as a horizontal cantilever beam. However, in the actual installation, the motor was mounted on a relatively flexible support which was not rigid. The motor manufacturer stated that to avoid excessive vibration, the reed critical frequency of the motor-pump system should be at least 25 percent above or below the operating speed. The complete system included the motor, pump, foundation, and piping.

Pump Impact Tests

Impact tests were also conducted to measure the mechanical natural frequencies of the pump. The pump was hung from a crane with the motor removed. The entire assembly with the motor attached could not be tested because the entire assembly could not be lifted with the motor lifting lugs. Accelerometers were located at the top of the pump and near the impeller.

The pump was impacted near the impeller and the resulting natural frequencies were 3.6 Hz (216 cpm) and 20.8 Hz (1248 cpm). These are natural frequencies of the pump in the free-free condition and these frequencies were changed when the motor was installed and the system was bolted to the foundation. When the pump was installed, the natural frequencies were reduced from 216 cpm to approximately 150 cpm and from 1248

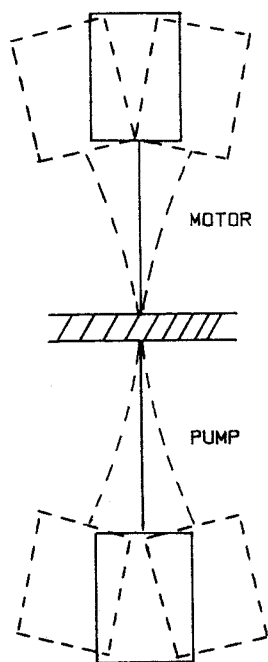


Figure 12. Reed Vibration Mode Shape.

cpm to approximately 900 cpm. The frequency reduction was due to the added weight of the motor.

The impact vibration mode shape agreed with the shaker data and the running data. The vibration amplitudes were larger on the motor end at both frequencies, and the ends were out-of-phase at the first frequency and in-phase at the second frequency.

Computer Model

The motor-pump system natural frequencies can be calculated using three-dimensional finite-element analysis as previously discussed by Corley [3] and Cornman [5]. In their analyses, the mounting plates were assumed to be fixed to the foundation at the anchor bolt location. However, the test results discussed herein have shown that the natural frequencies were sensitive to the effective stiffness of the attachment to the concrete and that the baseplates were not rigidly attached to the foundation.

A detailed computer analysis was not conducted, since the field tests indicated that stiffening the base corrected the vibration problem. It is felt that the system could be accurately analyzed using a detailed finite element model or a more simple lumped-mass model as shown in Figure 13. The model could be attached to the foundation with springs which represent the combined stiffness of the steel bases and the bolts. The neoprene pads essentially reduced the stiffness of this spring, while the gusset plates and the anchor bolts stiffened this spring.

Vibration Absorber

When a structure is operating near its mechanical natural frequency, it is often possible to reduce the vibration levels by adding an auxiliary mass on a spring tuned to the excitation frequency [6, 7]. The auxiliary spring-mass system is referred to by several names including dynamic absorber, damped absorber, vibration absorber, and detuner.

The design of an effective dynamic absorber is usually experimental, requiring additional testing to optimize the spring, mass, and damping. Also, the spring has to be designed for infinite fatigue life.

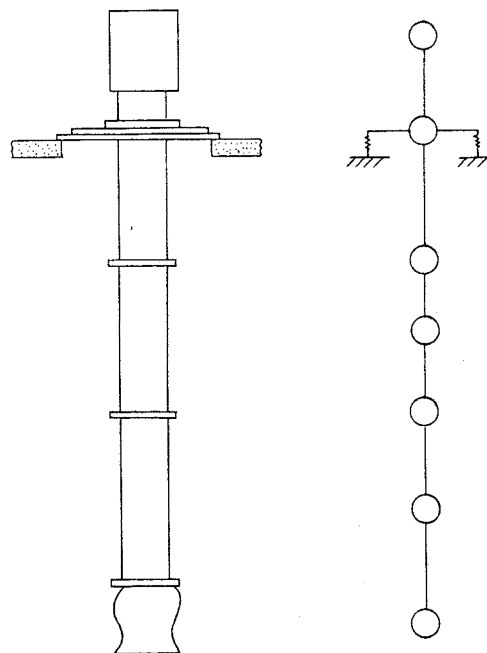


Figure 13. Lumped-Mass Representation of Motor-Pump System.

Dynamic absorbers were considered as a possible solution to the pump vibration problem, but were not tried, since the vibration problems were corrected by stiffening the pump mountings.

CONCLUSIONS

- The excessive vibrations and wear failures were due to operating near the motor-pump system mechanical natural frequency.
- The system mechanical natural frequency was very sensitive to the effective stiffness of the connection between the concrete, baseplates, pump base, and motor flange.
- The system natural frequencies were different in the N-S and E-W directions, due to the asymmetry of the baseplate.
- The rigid mounting resulted in lower overall vibration levels and was preferred to the soft mount with the neoprene isolation pad.
- The horizontal vibration levels near the top of the motor were proportional to the pump housing vibration near the impeller.
- The velocity probes installed near the impeller operated satisfactorily for a relatively long period in water and could be used during shop tests or acceptance tests to evaluate the vibration levels near the impeller.
- The combined motor-pump mechanical natural frequencies should be determined prior to installation. It is desirable for the system natural frequencies to be at least 20 percent from the running speed.
- Shop tests should be made with the pump mounted similarly to the actual installation. This means that the pump should be rigidly anchored to the exact baseplates and not temporarily mounted on rails or rubber pads.
- The system mechanical natural frequencies can be easily measured using a variable speed shaker. The shaker is especially useful for measuring natural frequencies above the running speed.

- The system natural frequencies can also be measured with impact tests, although it is sometimes difficult to test the pumps with high background vibration.

- Vertical pump vibration problems of the type described may often be solved through relatively minor structural modifications.

- Finite element models of vertical pump systems should have the capability of modelling the attachment stiffness.

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