

Vibration Analysis of Vertical Pumps

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Vibration data were collected on several large motor-driven vertical cooling water pumps which experienced excessive wear of the impellers, wear rings and seals after a short period of operation. The data indicated that the problem was due to the operating speed being near the pump-motor system mechanical natural frequency, which resulted in excessive vibration levels on the motors and pump impellers. A successful solution to the problem involved increasing the stiffness of the pump mountings.

Two months after installing four vertical single-stage, electric motor-driven pumps, all of the pumps experienced seal failures, bearing failures, and excessive wear on the impellers and wear rings. The pumps (Figure 1) were installed to provide cooling water to a refinery. The pump design data and operating conditions were as follows:

Design Flow 20,000 gpm
Minimum Intermittent Flow 4,000 gpm
Suction Pressure -10 psig
Differential Pressure 75 psig
TDH 174 feet
Submergence (4 feet req)
Speed 885 cpm
Horsepower Required 987 bhp

The seal failures allowed water to contaminate the bearing grease, which contributed to the bearing failures. When the seals were worn, the water forced the grease out of the top seal near the coupling, between the motor and pump. The impeller and shaft were primarily worn on one side, while the stationary parts indicated excessive wear completely around the circumference.

Mechanical Checks

Visual observation and vibration measurements at the top of the motor indicated that the motor vibration levels were excessive. The vibration levels at the top of the motor were often above 20 mils peak-to-peak, which exceeds the acceptable vibration limit established by the Hydraulic Institute of 6 mils for vertical pumps at 900 cpm.

It was not known whether the excessive vibration levels were related to wear problems on the pump. Consequently, a comprehensive program was developed to determine the mechanical integrity of the individual pumps. As part of that program, the following mechanical tests were performed:

- Several motors were operated uncoupled from the pump to obtain vibration readings for comparison with the previous data taken when the motors and pumps were coupled.
- Each coupling was checked for fit, concentricity, and dynamic balance.
- The pump and mounting heads were checked for parallel machining and level installation.
- The motor alignment to the pump was verified.
- The impellers were dynamically balanced.
- The shafts on the pumps were checked for straightness.
- The pump bases were removed and re-installed with proper shims.
- Pump internal clearances were checked.
- Additional lubrication points were installed to lubricate the pump shaft.

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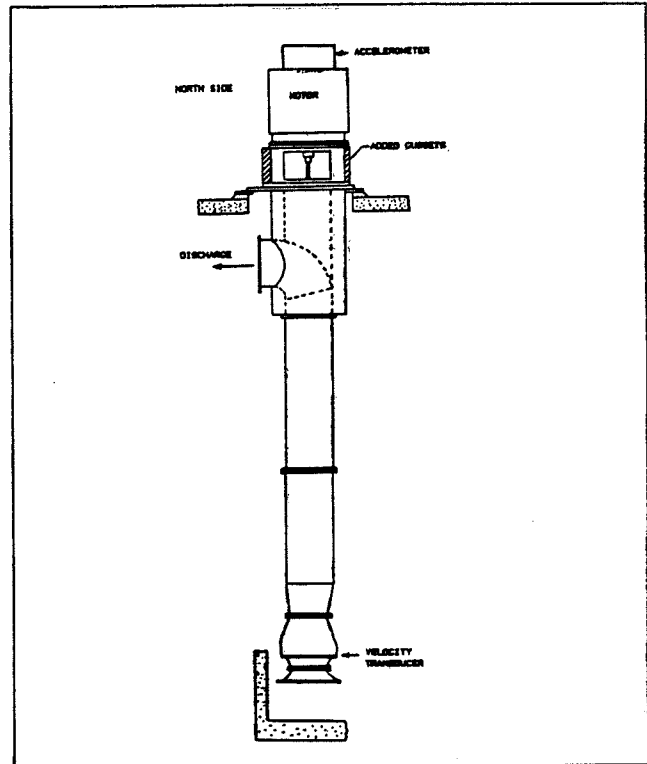


Figure 1. Vertical pump assembly.

Lubrication Problem

The mechanical tests revealed that the pumps were generally installed correctly and the dimensions and tolerances conformed to specifications by the pump and motor manufacturers. The tests did verify, however, that the pumps had a basic lubrication problem.

It was found that the grease system would not develop enough pressure to overcome the system back pressure. This problem resulted in frequent lubrication trip-outs, and the pumps would not receive proper lubrication until an operator checked them. Inspection of the bearings revealed that the grease could not pass through the bearings because of a lack of grease passages.

The lubrication problems were corrected by making the following changes:

- Installing grease pumps that maintained a constant flow of grease into the pump.
- Redesigning the bearings with longitudinal grease passages.
- Adding a third grease point just above the fourth bearing from the top.

Although the modifications improved the pump lubrication, the seal and impeller failures were not eliminated. This suggested that the problem was caused by the design rather than incorrect installation or maintenance techniques.

Sump Vortex

Initially, the pump manufacturer suspected that the pump vibration could be the result of vortices in the sump. Visual observation of the water surfaces in the sump showed possible vortex circulation at several locations.

Vortices at pump intakes are a well known source of problems, including: 1. reduced pump efficiency, 2. vibration and noise, 3. increased bearing wear, and 4. accelerated deteriora-

tion of impeller blades caused by abrasion from entrained debris, corrosion due to excessive air, and pitting.

A review of the pump design indicated general conformance with the Hydraulic Institute guidelines. Although the sump design appeared satisfactory, the pump manufacturer believed a more detailed analysis of the sump should be made.

Time constraints did not permit the detailed sump analysis to be performed, however. A sump analysis can typically require several months to complete. Instead, the manufacturer installed a vortex suppressor (vortex splitter in the inlet strainer) in an effort to eliminate a possible vortex at the pump inlet.

Vibration data recorded on the pump with the vortex suppressor indicated the vibration levels were not reduced. While there may have been evidence of vortices in the sump, it was concluded that the excessive pump vibration was not caused by vortices at the pump inlet.

Vibration Transmission

The pumps were mounted on a common concrete structure and discharged into a common header, which provided transmission paths for the vibration between the pumps. It was observed that the motor vibration amplitudes would significantly increase and decrease every few minutes (beating phenomenon) as the vibration of the various pumps were in phase and out of phase.

In an effort to reduce the vibration transmission between the pumps and the discharge piping, a rubber bellows was installed in the discharge piping of one of the pumps. The vibration levels on the pump were not reduced, which meant that isolating the discharge piping would not solve the discharge problem.

Vibration Monitoring

The user company requested consulting engineers to assist in determining the causes of the problems and recommend modifications to solve the problems. In recent years, it has become fairly common practice to install vibration monitoring systems on large machinery. However, vertical pumps, such as cooling water pumps, are generally not equipped with vibration transducers.

It is difficult and expensive to install underwater instrumentation to measure vibrations near the impeller. Generally, vibrations are monitored near the top of the motor, because it has been found that motor vibration can sometimes be indicative of vibration on the impeller.

To obtain vibration data on the pump column, the pump manufacturer installed two velocity transducers near the impeller to measure vibration in the East-West (E-W) direction, perpendicular to the discharge flow, and in the North-South (N-S) direction, parallel to the flow (Figure 1). The transducers

were mounted on a pump before it was reinstalled in the sump.

For underwater service, velocity probes are advantageous to other types of transducers, such as accelerometers and proximity probes, because the velocity transducers generate an electrical signal without a power supply or signal conditioner. The velocity probes operated successfully in the river water for several months without any special waterproofing.

Ideally, vibration data should be taken at several locations between the impeller and the top of the motor to accurately define the vibration mode shape. However, these test points were inaccessible and, for safety reasons, personnel were not permitted in the sump when the pumps were operating. Therefore, vibration measurements were limited to areas above the top of the deck and the velocity transducers installed on the one pump.

Vibration Versus Discharge Pressure

Operation with several pumps in service raised the system discharge pressure and increased vibration levels and beating between the pumps. Therefore, tests were performed to determine whether the increased vibration levels were directly related to the increased discharge pressure.

Due to operational constraints, the system discharge flow rates and pressure could not be tested over a wide range. Consequently, it was decided to increase the pressure on individual pumps by throttling the block valve at the pump discharge.

The flowrates were estimated using pump performance curves and motor horsepower. The motor horsepower was calculated from the measured motor amperage.

Tests were made on two pumps: G-8405, which was recently overhauled and thought to be in good mechanical condition, and G-8403, on which velocity probes had been installed near the impeller. As shown in Table 1, the vibration amplitudes remained fairly constant for the range of discharge pressures tested.

The measured total head versus horsepower curves generally agreed with the curve supplied by the manufacturer. Pump G-8405 was in better mechanical condition and performed closer to the design curve, compared to pump G-8403, which had higher vibration levels and increased seal clearances. This test indicated the pumps were operating near the design flow condition, and that the vibration levels were not significantly affected by the discharge pressure.

Structural Mechanical Natural Frequency

A variable speed mechanical shaker attached to the top of the pump motor was used to excite the motor-pump mechanical natural frequencies (Figure 2). The shaker speed was varied from approximately 400 cpm to 1200 cpm to identify the

Table 1. Pump vibration versus discharge pressure.

Discharge pressure psig	Total Head ft	Pump G-8405					
		Amps	Motor Load HP	Motor Vibration			
				E-W mils	N-S mils		
60	147.4	127	1021	7-10	4-5		
65	159.0	125	1005	7-9	5		
70	170.5	122	981	6-7	2-3		
75	182.0	117	940	6-7	3-4		
80	193.6	108	868	7-9	5-6		
Pump G-8403							
Discharge pressure psig	Total Head ft	Motor Load		Motor Vibration		Impeller Vibration	
		Amps	HP	E-W mils	N-S mils	E-W in./sec	N-S in./sec
60	147.4	123.5	993	11.4	7.3	0.24	0.26
65	159.0	121.0	973	11.8	5.5	0.26	0.20
70	170.5	117.0	940	14-15	3-5	0.30	0.17
75	182.0	109.0	876	13.5	2-5	0.32	0.17-0.20
80	193.6	104.0	836	12.5	2-5	0.28-0.30	0.14-0.23

Total Head = 2.307 × (psig) + 9 ft. (distance of gage to water)

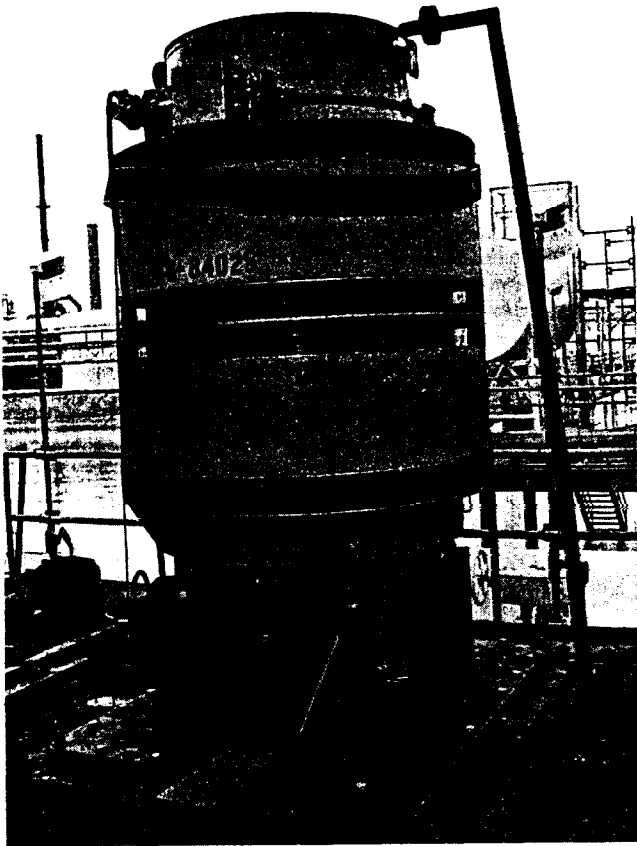


Figure 2. Variable speed shaker mounted on the motor.

mechanical natural frequencies. Vibration levels at the shaker running speed and the vibration phase angle relative to the shaker keyphasor signal were plotted in Bodé format versus shaker speed. The data were also plotted in polar format with the amplitudes plotted in 1/4 scale.

The mechanical natural frequencies were measured on several units, but most of the testing was done on pump G-8403. As shown in Figures 3 and 4, the mechanical natural frequency in the E-W direction was 870 cpm on the top of the motor and near the impeller. This test illustrated that the response at the top of the motor was proportional to the vibration at the impeller and was not a response of the motor alone.

Attachment Stiffness

Testing showed that the mechanical natural frequency was a direct function of the stiffness of the bolted connections between the concrete, pump, and motor. To illustrate this effect, the anchor bolts were loosened and the natural frequency was reduced from 870 cpm to 840 cpm (Figure 5). The vibration amplitude at the pump running speed was reduced from 45 mils to 35 mils, because the response was further removed from the pump running speed.

All of the bolts were then re-tightened and the resonance was increased from 840 cpm to 900 cpm (Figure 5). The motor vibration amplitude at the pump running speed increased to 60 mils. These tests indicated that the vibration amplitudes at the pump operating speed were actually lower with the anchor bolts loose, because the natural frequency was reduced to 840 cpm.

The pump base plate was rectangular, which placed the anchor bolts further from the pump in the N-S direction. This increased distance to the anchor bolts made the base more flexible in the N-S direction, compared to the E-W direction. The increased flexibility lowered the mechanical natural frequency in the N-S direction further below the pump running speed, which explains why the vibration amplitudes were

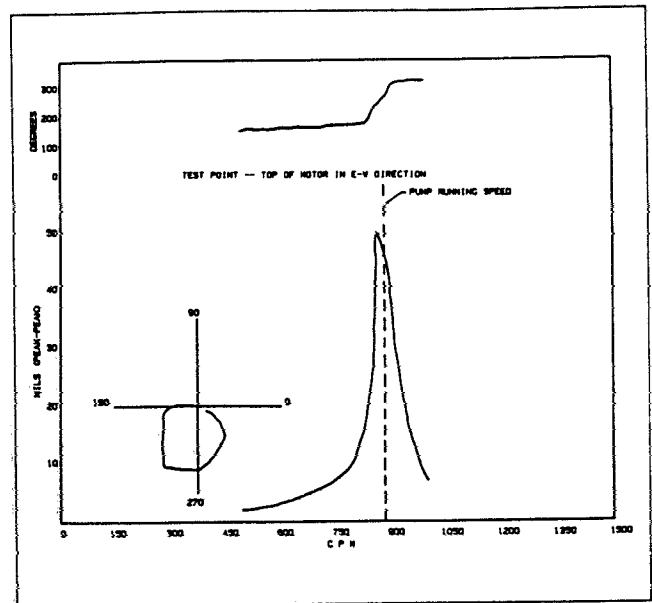


Figure 3. Natural frequency of original system - measured at the top of the motor.

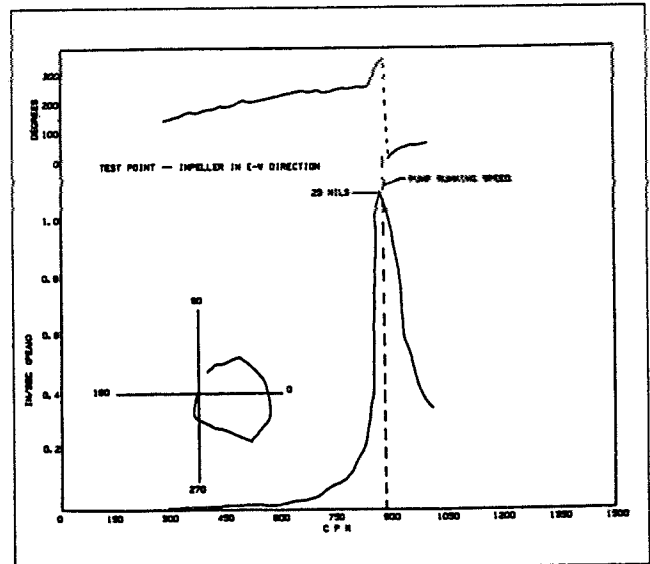


Figure 4. Natural frequency of original system - measured at the impeller.

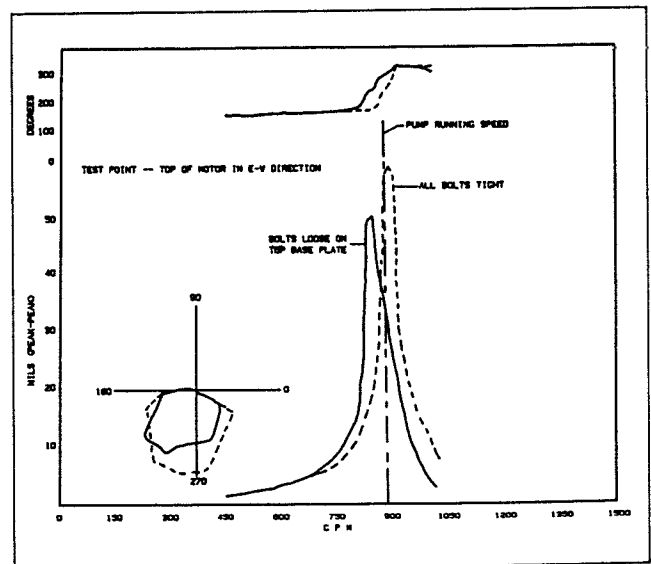


Figure 5. Effect of anchor bolt torque on natural frequency.

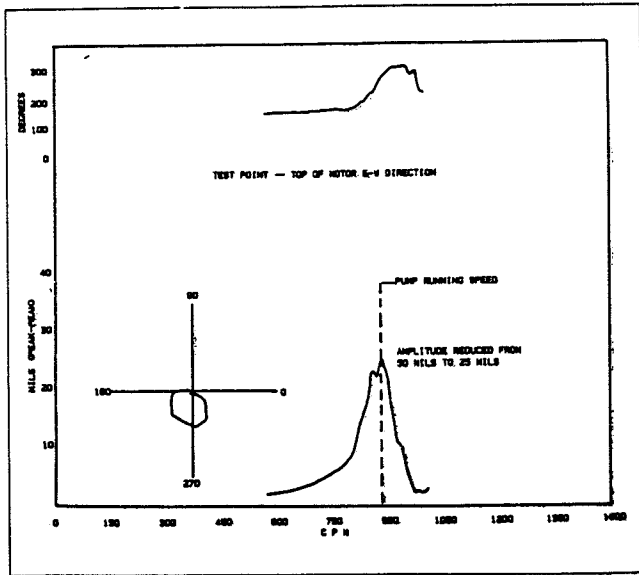


Figure 6. Effect of temporary brace - measured at the top of the motor.

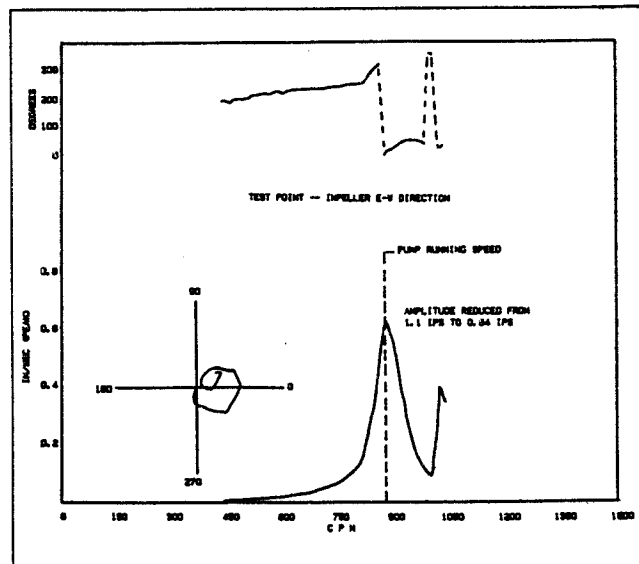


Figure 7. Effect of temporary brace - measured at the impeller.

generally lower in the N-S direction.

These tests confirmed that the response of the baseplate was very sensitive to the effective stiffness of the connection to the concrete. The effective attachment stiffness varied considerably between each of the pumps, due to the tightness of the anchor bolts and the alignment shims. This explained the difference in the mechanical natural frequencies between "identical pumps" and why some of the pumps were more vibration sensitive.

Temporary Bases

Field data indicated the pumps were vibrating individually and were not necessarily vibrating in relation to the phase of adjacent units. One method often used to increase the stiffness of a system is to physically tie the individual units together. To evaluate this effect, a portable hydraulic jack was used to wedge a four-inch by four-inch wooden timber between the tops of two adjacent motors.

The timber reduced the vibration amplitudes from 50 mils to 25 mils at the top of the motor and from 1.1 ips (23 mils) to 0.64 ips (13.8 mils) at the impeller, as shown in Figures 6 and 7. The reduced vibration appeared to be primarily the result of increasing the damping in the system rather than increasing the system stiffness because the natural frequency was not significantly changed.

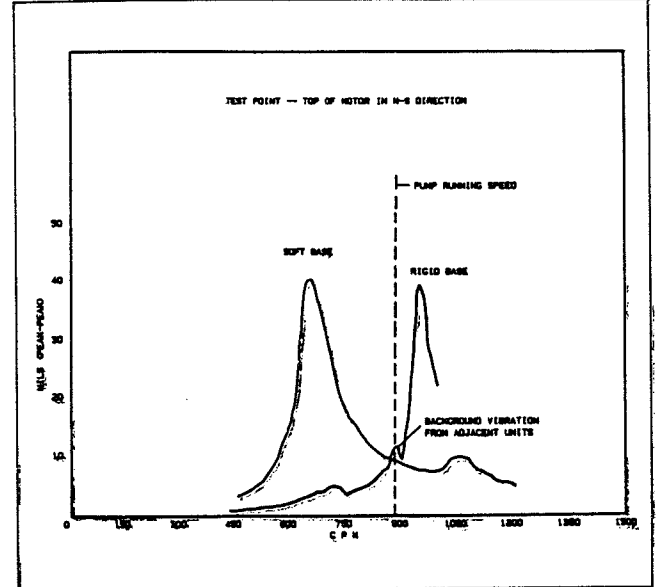


Figure 8. Comparison of motor response in the N-S direction with soft and rigid bases.

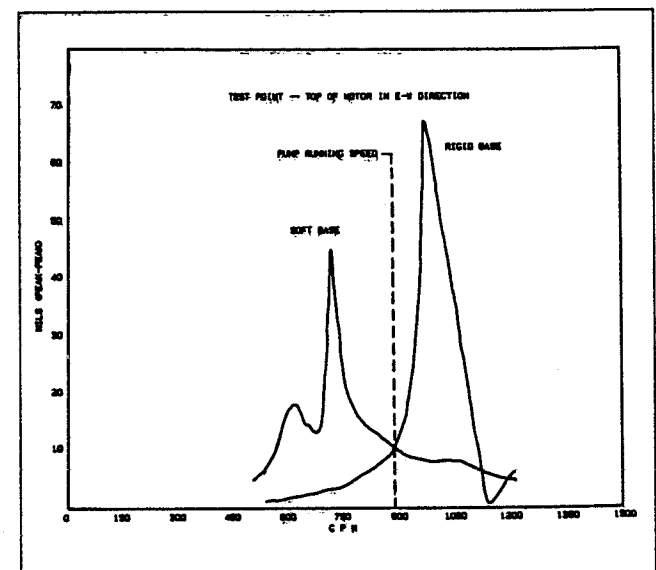


Figure 9. Comparison of motor response in the E-W direction with soft and rigid bases.

This test indicated it would be possible to reduce the motor and impeller vibration levels by tying the units together. While the braces would have provided a temporary solution, they were not recommended for the following reasons:

1. It would have been difficult to attach braces to the motor.
2. The pumps on the ends were adjacent to shorter diesel-driven units used for emergency service and, thus, could be braced only on one side.
3. The motor manufacturer would not approve the braces as a long-term modification.

Mechanical Natural Frequency Modification

Tests demonstrated that the vibration levels could be reduced by moving the natural frequency further from the running speed. The natural frequency could be lowered by mounting the pump on a soft base, or the frequency could be raised above the running speed by rigidly attaching the upper baseplate to the lower baseplate and stiffening the motor mounting base.

Soft Base. Neoprene isolator pads were installed between the upper and lower base plates to decrease the pump-motor system mechanical natural frequency further below the pump running speed. Anchor bolts, which attached the upper and

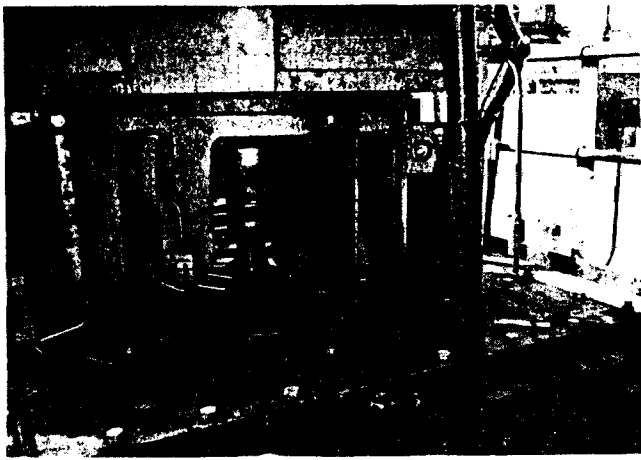


Figure 10. Gusset plates and additional anchor bolts.

lower base plates, were isolated from the bases with rubber sleeves and rubber washers to ensure there was no mechanical coupling between the bases. The bolts were "finger tight" to avoid crushing the pad.

Shaker tests revealed the system mechanical natural frequency was lowered to approximately 660 cpm in the N-S direction and 720 cpm in the E-W direction (Figures 8 and 9). Vibration amplitudes at the pump running speed were reduced by a factor of approximately six-to-one on both the motor and impeller.

Vibration data recorded with all the motor driven pumps running showed that the motor vibration at the pump running speed was reduced from 13.7 mils to 2 mils in the N-S direction and from 3.3 mils to 2.8 mils in the E-W direction. Although the vibration amplitudes were generally reduced at the pump running speed, the beating between the other pumps was still present. Also, the soft mounting made the system more sensitive to random low-level turbulence, which caused increased vibration at the system natural frequencies.

Rigid Base. Additional bolts were added to rigidly attach the upper base to the lower base plate. Gusset plates were temporarily installed to stiffen the motor base (Figure 10). The system natural frequencies were increased to 950 cpm in the N-S direction and 975 cpm in the E-W direction (Figures 8 and 9). While the natural frequency was still within nine percent of the running speed, the amplitudes at the pump running speed were reduced approximately four-to-one.

Data recorded with the pumps in operation indicated that the vibrations were primarily occurring at the pump running speed. The vibration amplitudes were reduced by a factor of 5.0 in the E-W direction and a factor of 2.5 in the N-S direction, compared to the data before the modification. The amplitude reduction favorably agreed with the shaker data, which indicated a four-to-one reduction.

Analysis of Soft System

Advantages. The isolation pad was simple to install and did not require structural modifications. The natural frequency was lowered approximately 180 cpm (20 percent below the running speed), which reduced the running speed vibration. Also, the pad increased the isolation from structure-borne vibration.

Disadvantages. The system had low stiffness, and the vibration produced by random turbulence caused the vibration at the system natural frequencies to be a significant portion of the overall total vibration. Also, the vibration at running speed, caused by unbalance, could be increased because the system was more compliant, or less stiff. A potential problem existed, in that the neoprene could deteriorate and lose its elastic properties, which would allow the natural frequency to move closer to the running speed. Installation of the isolation pads was also difficult, requiring a large crane to lift the entire pump assembly to install the pads.

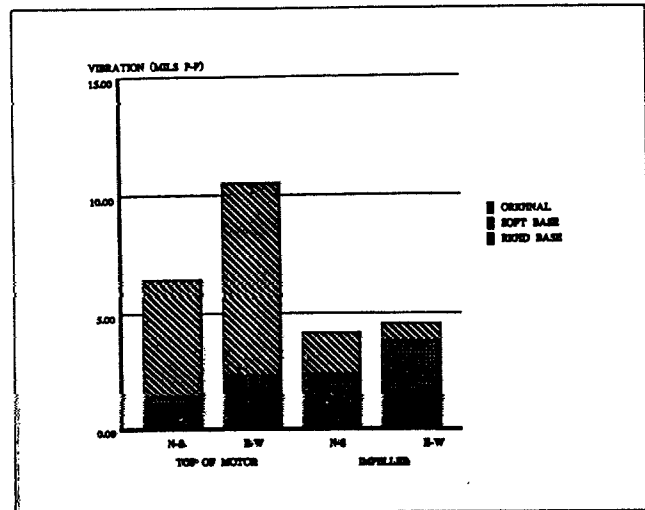


Figure 11. Comparison of overall vibration levels with different bases.

Analysis of Stiff System

Advantages. The stiff system offered several advantages in comparison to the soft system. The system natural frequency was effectively increased above the running speed, which reduced the vibration at running speed. Increasing the natural frequency above running speed also prevented the natural frequency from being excited during startups and shutdowns. Another advantage was that the system was stiffer and less sensitive to low level turbulence and increases in unbalance. The extra anchor bolts and gussets also could be installed without removing the pumps from their bases, which was an important consideration from a maintenance point of view.

Disadvantages. The gusset plates and anchor bolts were costly to install, in comparison to the isolation pads. The company chose to temporarily attach the gussets with bolts, to avoid a potential alignment problem. Also the anchor bolts were drilled and tapped, which increased the costs and installation time. The bolting created potential problems, because the natural frequency could be lowered closer to running speed if the bolts loosened. This problem could be avoided, however, by welding the gussets and baseplates instead of using bolts to attach the gussets.

Preferred Solution

The company conducted additional tests to accurately evaluate the vibration characteristics with the soft and hard mountings. The data indicated that while the soft mounting reduced the overall vibration amplitudes compared to the original design, the vibration levels with the rigid mounting were even further reduced (Figure 11).

Therefore, the preferred solution was to stiffen the base. All of the pump bases were similarly stiffened, and the pump vibration levels were reduced. More importantly, wear failures were eliminated. The pumps have been in service for approximately one year with the stiffened bases, and there have been no reported failures from excessive vibration.

Further Tests and Modelling

The term "reed critical frequency" is often used in the pump industry when referring to the mechanical natural frequency of vertical pumps and motors. In the past, many people have analyzed the motor-pump system as two different independent systems with the motor and pump individually attached to a rigid mass (Figure 12).

The motor manufacturer quoted the motor reed critical frequency to be 1650 cpm for the motor bolted to a rigid support and considered as a horizontal cantilever beam. However, in the actual installation, the motor was mounted on a relatively flexible support that was not rigid. The motor manufacturer stated that to avoid excessive vibration, the reed critical frequency of the motor-pump system should be at least 25

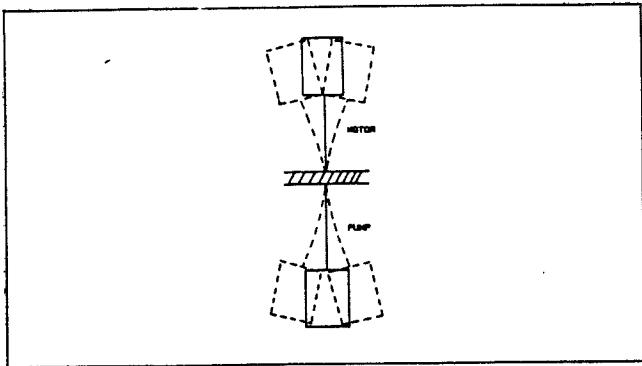


Figure 12. Reed vibration mode shape.

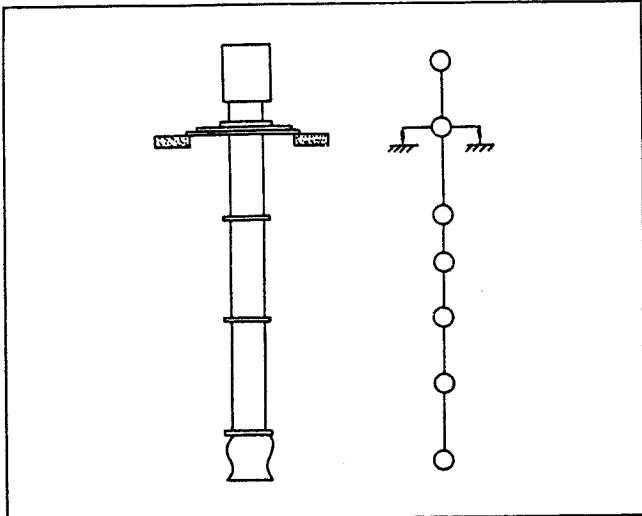


Figure 13. Lumped-mass representation of motor-pump system.

percent above or below the operating speed. The complete system included the motor, pump, foundation, and piping.

Impact tests were also conducted to measure the mechanical natural frequencies of the pump. The pump was hung from a crane with the motor removed. The entire assembly with the motor attached could not be tested because the entire assembly could not be lifted with the motor lifting lugs. Accelerometers were located at the top of the pump and near the impeller.

The pump was impacted near the impeller and the resulting natural frequencies were 3.6 Hz (216 cpm) and 20.8 Hz (1248 cpm). These are natural frequencies of the pump in the free-free condition, and these frequencies were changed when the motor was installed and the system was bolted to the foundation. When the pump was installed, the natural frequencies were reduced from 216 cpm to approximately 150 cpm and from 1248 cpm to approximately 900 cpm. The frequency reduction was due to the added weight of the motor.

The impact vibration mode shape agreed with the shaker data and the running data. The vibration amplitudes were larger on the motor end at both frequencies, and the ends were out-of-phase at the first frequency and in-phase at the second frequency.

Computer Model

The motor-pump system natural frequencies can be calculated, using three dimensional finite-element analysis as previously discussed by Corley¹ and Cornman². In their analyses, the mounting plates were assumed to be fixed to the foundation at the anchor bolt location. However, the test results discussed herein have shown that the natural frequencies were sensitive to the effective stiffness of the attachment to the concrete and that the baseplates were not rigidly attached to the foundation.

A detailed computer analysis was not conducted, since field tests indicated that stiffening the base corrected the vibration

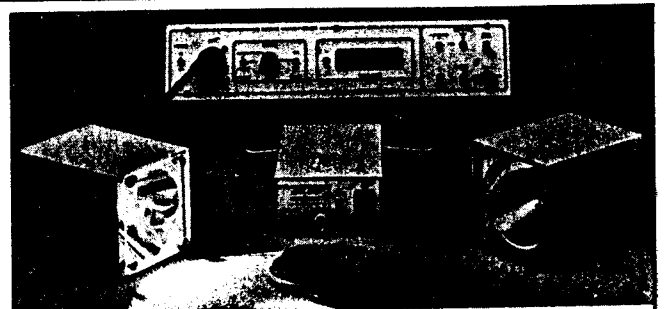
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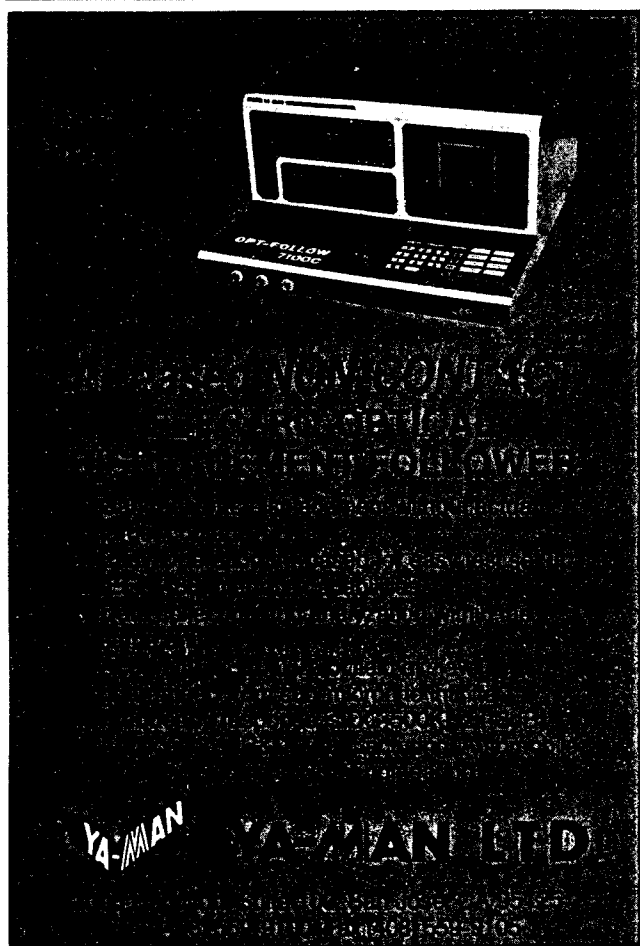
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problem. It is believed that the system could be accurately analyzed, using a detailed finite element model or a more simple lumped-mass model as shown in Figure 13.

The model could be attached to the foundation with springs, which represent the combined stiffness of the steel bases and bolts. Neoprene pads essentially reduced the stiffness of this spring, while gusset plates and anchor bolts stiffened the spring.

Vibration Absorber

When a structure is operating near its mechanical natural frequency, it is often possible to reduce the vibration levels by adding an auxiliary mass on a spring tuned to the excitation frequency.^{3,4} The auxiliary spring-mass system is referred to by several names, including dynamic absorber, damped absorber, vibration absorber, and detuner.

The design of an effective dynamic absorber is usually experimental, requiring additional testing to optimize the spring, mass, and damping. Also, the spring has to be designed for infinite fatigue life.

Dynamic absorbers were considered as a possible solution to the pump vibration problem, but were not tried, since the vibration problems were corrected by stiffening the pump mountings.

Conclusions

Experience with these vertical pumps added to our knowledge of their behavior characteristics as well as effective testing methods. From a behavioral standpoint, excessive vibrations and wear failures were caused by operating near the motor-pump system mechanical natural frequency. The system mechanical natural frequency was very sensitive to the effective stiffness of the connection between the concrete, baseplates, pump base, and motor flange. The system natural frequencies were different in the N-S and E-W directions, due to the asymmetry of the baseplate. Rigid mounting resulted in lower overall vibration levels and was preferred to the soft mount with the neoprene isolation pad.

From a testing standpoint, velocity probes installed near the impeller operated satisfactorily for a relatively long period in water and could be used during shop tests or acceptance tests to evaluate vibration levels near the impeller. It is also advantageous to determine the combined motor-pump mechanical natural frequencies prior to installation. It is desirable for the system natural frequencies to be at least 20 percent from the running speed.

Shop tests should be made with the pump mounted similarly to the actual installation. In the case illustrated in this article, the pump should be rigidly anchored to the exact baseplates and not temporarily mounted on rails or rubber pads.

System mechanical natural frequencies can be easily measured using a variable speed shaker. The shaker is especially useful for measuring natural frequencies above the running speed. The system natural frequencies can also be measured with impact tests, although it is sometimes difficult to test the pumps with high background vibration.

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