Torsional Analyses of Variable Frequency Drives

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Introduction

One of the major improvements in efficiency of plant operation has been the use of variable frequency driven motors so that the motor speed can be adjusted to maximum efficiency in the system. Due to the increased operating speed range and additional excitation mechanisms, variable frequency motors require special consideration when analyzed for torsional vibrations.

The torsional analysis performed for motors utilizing variable frequency drive (VFD) controllers includes the evaluation of the system response to dynamic torques by both the electrical excitation at harmonics of the electrical frequency resulting from the variable frequency drive and the mechanical excitation at the operating speed. The electrical excitation frequencies of concern include the fundamental electrical frequency, and the 6^{th} and 12^{th}orders of electrical frequency.

The torsional response characteristics of rotating equipment should be analyzed and evaluated to ensure the system's reliability. Severe torsional vibrations often occur with the only indication of a problem being gear noise or coupling wear. Excessive torsional vibrations can result in gear wear, gear tooth failures, key failures, shrink fit slippage and broken shafts in severe cases. Specifications such as API 617, API 618 and and U.S. MIL STD 167 provide guidelines and criteria for evaluating system torsional response characteristics. The *Shock and Vibration Handbook* [1], Nestorides [2], Ker Wilson [3], and *Rotordynamics of Machinery* [4] provide references on torsional analysis procedures. The two major analysis techniques commonly used are the Holzer Method and the Eigenvector-Eigenvalue procedure (Modal Superposition Method).

In performing a torsional analysis, the first step taken by Engineering Dynamics Incorporated (EDI) is to calculate the torsional natural frequencies and mode shapes utilizing an Eigenvector-Eigenvalue Method. The calculated natural frequencies are then compared to the excitation frequencies such as running speed and its multiples to determine if the system satisfies the appropriate specifications. For example, API 617 specifies that the torsional modes of the complete unit should be at least 10 percent below any operating speed or at least 10 percent above the trip speed.

After the torsional natural frequencies are calculated, forced vibration response calculations are performed to calculate the shaft torsional stresses. If the system does not meet the proposed criteria, a parametric analysis can identify sensitive elements which may be adjusted to modify the torsional responses and make the system acceptable.

If resonances cannot be avoided, coupling selection can be optimized based on torsional shaft stress calculations, as well as location of critical speeds. When changes in coupling specifications result in a coupling that has a different weight than the vendor originally specified, the lateral critical speeds may be affected. Heavier couplings will lower the lateral critical speeds while lighter couplings will raise them. The lateral critical speeds should be at least 20 percent from the operating speed.

EDI's torsional analysis procedures are discussed in greater detail in the following sections.

Steady-State Analysis

Modeling

To simulate the torsional response of an entire system, the rotating masses are represented by a series of lumped torsional inertias connected by torsional springs which represent the shaft stiffnesses between the torsional mass inertias.

Torsional Inertias

Each significant polar mass moment of inertia should be considered in the analysis. Typical inertias include motor and generator rotors, coupling hubs, compressor or pump impellers, and gears. Each vendor normally provides the torsional mass moment of inertia values for their equipment components. These inertia values should be checked prior to performing the computer analysis. Formulas for calculating the polar moment of inertia for selected configurations are given in Figure 1. The polar moments of inertia are sometimes called WR^2 values, where R is defined as the radius of gyration.

Torsional Stiffnesses

Shafts

The torsional stiffnesses between masses (torsional inertias) are normally calculated from dimensioned shaft drawings. The stiffness of a uniform diameter shaft is equal to I_pG/L where I_p is the polar area moment of inertia, L is the shaft length, and G is the shaft material shear modulus. An equivalent shaft stiffness is calculated for shaft sections with diameter changes between masses. The effective spring constant (K_e) is the series equivalent and is determined by $\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \ldots$ where K_1, K_2, and K_3 are the spring constants for the shafts of diameter D_1, D_2, and D_3. If the effective shaft stiffnesses are provided by the vendor, they should be verified by calculations whenever possible. The equations for calculating the torsional stiffnesses of several cross sectional shapes are given in Figure 2.

Couplings

Couplings are typically modeled with a torsional inertia at each hub connected by the coupling stiffness. The torsional stiffness of couplings obtained from the manufacturer is normally used since experience indicates good correlation using their data. "Certified coupling drawings" from the vendor are recommended to ensure that the torsional analysis is made with the correct couplings. The vendor's coupling stiffness usually assumes one-third penetration of the shaft into the hub, which means that two-thirds of the shaft length in the hub has no relative motion. The stiffnesses of the adjacent shaft sections are thus simulated up to the edge of the coupling hub, as illustrated in Figure 3. This assumption should be confirmed with the vendor prior to modeling the system. The stiffness contribution of shrink or hydraulically fitted coupling hubs can be analyzed for specific cases.

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Some couplings have a constant torsional stiffness independent of speed or load, while other couplings utilize nonlinear elastic elements which add damping to the system. These couplings with added damping serve to reduce the peak amplitudes at resonance. The elastic elements cause a change of torsional stiffness which varies with load and speed. At each speed, the coupling torsional stiffness must be calculated for the transmitted torque.

Manufacturers usually specify the allowable vibratory torque values for their couplings. The torsional analysis predicts the calculated dynamic torque on the coupling across the speed range, and this can be compared to the allowable specified by the manufacturer.

Gear Teeth

In most systems the equivalent gear mesh torsional stiffness is high compared to the torsional shaft stiffnesses and has little influence on the lower torsional natural frequencies. In most field problems the lower torsional natural frequencies are the ones of most concern. However, the gear tooth stiffness still should be correctly modeled to accurately predict all the torsional natural frequencies. The gear mesh stiffness can have a significant effect in systems with multiple gear sets.

The gear tooth stiffness can be calculated from the gear tooth geometry by considering the effects of tooth bending, tooth shear deflection and surface compression.

Geared Systems

The torsional analysis programs are able to simulate multi-branch geared systems which may occur in compressor trains, marine propulsion systems or engine transmissions. The most common method of modeling systems of this type is to generate an equivalent system. In the equivalent system, the shaft stiffnesses and torsional inertias are adjusted so that the proper natural frequencies are calculated. The torsional inertias and stiffnesses of one branch are multiplied by the speed ratio squared. ($I_2' = I_2'(N_2/N_1)^2$, $K_2' = K_2(N_2/N_1)^2$ where N_2 and N_1 are the speeds of branches 2 and 1 respectively.) If the gear teeth are considered rigid, the equivalent pinion and bull gear inertias can be combined. However, the actual system including gearing effects (tooth stiffness and individual gear and pinion inertias) should be modeled. This is particularly important in high speed systems where excitation frequencies could be near the modes sensitive to tooth stiffness variation, or in systems with multiple gear sets or branches.

Calculation of Torsional Natural Frequencies and Mode Shapes

The torsional natural frequencies and vibration mode shapes can be calculated using an eigenvector-eigenvalue matrix solution technique (modal superposition) which directly solves the differential equations of motion for the lumped mathematical model of the torsional system. The differential equations for torsional system can be easily developed as illustrated in Figure 4 which shows how a nine inertia system is handled.

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In matrix notation, the system of differential equations including damping is:

$$[J]\ddot{\theta} + [C]\dot{\theta} + [K]\theta = 0$$

where

$J =$ diagonal inertia matrix

$K =$ stiffness matrix

$C =$ damping matrix

$\theta =$ angular displacement vector, radians

$\dot{\theta} =$ angular velocity vector, radians/sec

$\ddot{\theta} =$ angular accelerations vector, radians/sec^2

Normally, the damping in torsional systems is small and has little effect on the natural frequencies. Therefore, the torsional natural frequencies and mode shapes may be calculated assuming the damping is zero. This makes the damping matrix $[C] = 0$ in the above equation. The solution of these equations can be obtained by assuming simple harmonic motion.

$$\theta = A \ \sin \ \omega t$$

The matrix equation can then be reduced to $[J][\omega^2]\theta = [K]\theta$ where ω^2 represents the diagonalized eigenvalue matrix. This form of the matrix equation is the eigenvalue equation which is solved for the eigenvalues and eigenvectors. The eigenvalues are the torsional natural frequencies and the eigenvectors are the vibrational mode shapes corresponding to each eigenvalue. Each set of eigenvectors is normalized with the maximum amplitude equal to 1.0 and plotted to illustrate the vibration mode shapes. The stiffness of non-linear couplings change with load and speed, and may result in some slight changes of the mode shapes.

Torsional natural frequencies of some simple systems can be calculated using the formulas given in Figure 5. Another method suitable for hand calculation is the Holzer method which is an iterative procedure for calculating torsional natural frequencies and mode shapes.

Avoidance of Critical Speeds

Systems which incorporate VFD controllers require additional considerations in the design stage over conventional constant speed equipment. The wide speed range increases the likelihood that at some operating speed a coincidence between a torsional natural frequency and an expected excitation frequency will exist.

Interference Diagram

The overall system reliability depends upon the location of the torsional natural frequencies with regard to the potential excitation frequencies. An interference diagram (Figure 6) is

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generated for each system to identify coincidences between expected excitation frequencies and torsional natural frequencies within the operating speed range. Whenever practical, the coupling torsional stiffness and inertia properties are selected to avoid any interferences within the desired speed range; however, this is usually not possible with VFD systems.

If a torsional natural frequency occurs within the desired speed range, several methods can be used to change the natural frequency. Mechanical tuning can be accomplished most easily by changing the coupling stiffness which usually controls the first natural frequency. Changes of shaft diameter or length can also shift the natural frequencies. It is normally difficult to change masses; however, a reduction of mass tends to raise the natural frequencies. If the critical speeds cannot be removed from the desired speed range, they should be avoided by a 20% margin, if feasible. If avoidance is not possible, a detailed stress analysis should be performed to determine if the stresses are within the appropriate engineering criteria.

Forced Vibration Response

The torsional response to excitation torques at any or all mass inertia locations can be accurately calculated using an eigenvalue-eigenvector computer program. The phasing of each forcing function frequency can be considered by using a complex Fourier expansion of the excitation torques.

Excitation Torques

Motors

The stepped voltage waveform produced by the VFD generates harmonic currents which cause the motor to generate torque modulations at 6, 12, 18, ..., times the fundamental electrical frequency. The magnitudes of the torque modulations are affected by the resonant frequency of the electrical L-C filter. The dynamic torques generated by the motor as a result of the variable frequency controller (6×, 12× electrical frequency) are obtained from the manufacturer during the design procedure for the variable frequency controller.

The fundamental electrical frequency is assumed to have a maximum variation in transmitted torque of one percent zero–peak. Although, theoretically, there should not be any net dynamic torque component at the fundamental electrical frequency for a 3 phase motor, it is not unheard of and has been documented in torsional tests. A value of 1% 0–p has been assumed and is considered conservative for large HP industrial motors. Similarly, torque modulations may be produced by mechanical sources in the system. The dynamic torques from mechanical sources in the system are assumed to be one percent zero–peak of the steady-state transmitted torque at the operating speed.

The analysis applies the torque from each excitation source at its frequency, and computes a resultant stress by combining all stress values from each component.

Centrifugal Pumps and Fans

In centrifugal pumps and fans driven by electric motors, torsional excitation at the equipment running speed can result from turbulence or load variations, but is generally less than 1 percent zero–peak of the steady state torque. Torque modulation from good quality gears is also usually less than 1 percent zero–peak at one times (1×) running speed; however, larger percentages have been measured on worn gearsets and bevel gears. Therefore, for 1× running speed, a 1 percent torque excitation (zero–peak) is used to calculate the torsional deflections and stresses. The excitation is typically applied at the inertia with the maximum calculated amplitude at that frequency to obtain the maximum calculated stress. Torsional excitation can also be applied at other potential excitation locations on the driver, the gear, and the driven equipment. If torsional natural frequencies coincide with multiples of running speeds, the torsional stresses are calculated with an assumed excitation level of 1/n percent where n is the harmonic. For cases where blade passing frequency coincides with torsional natural frequencies, an excitation level of 0.5 percent of steady state torque is used.

Damping

The torsional system damping can be modeled as modal damping, i.e., a damping value for each natural frequency, or discrete damping of any particular element can be simulated, such as torsional dampers or rubber block type couplings (Holset).

The system damping is related to the resonant amplification factor, Q, by the equation $Q = 1/2\zeta$ where ζ is the critical damping ratio. The Q value defines the sharpness of the resonant response. Q values of couplings with elastic elements can be as low as 4, while values of 30 are typical of gear type couplings. This corresponds to a range of damping values from 12.5 to 1.67 percent of critical damping.

Modal damping, derived empirically from numerous field measurements on machinery system is utilized with the applied forcing torques to calculate the torsional amplitudes and stresses.

Torsional Stress

The torque modulations produced by the prime mover, or loaded equipment are applied at the corresponding masses and the resultant stresses are calculated. The torsional stresses in the shafts are dependent upon the torsional excitation, the torsional natural frequencies, acceleration, and the system damping characteristics near the resonances. The shaft stresses are calculated assuming steady-state conditions; that is, the system operates continuously at each (plotted) speed. A steady-state torsional stress analysis is made at each resonant frequency and at interim speeds to plot the stresses as a function of speed.

The torsional shear stresses are calculated for each shaft based upon the relative twist in the shaft between two adjacent torsional mass inertias. The relative deflections are computed from the eigenvectors for each frequency component and recombined to obtain overall peak-to-peak amplitudes from the forced vibration response. The governing equation for calculating the stresses is:

$$S = \frac{\mathrm{SCF}\; G\; D\; \Delta\theta}{2L}$$

where:

$S =$ shear stress, lb/in^2

$G =$ shear modulus, lb/in^2

$D =$ minimum shaft diameter, in.

$\Delta\theta =$ differential torsional deflection, radians

$L =$ shaft length between torsional masses, in.

SCF = stress concentration factor, dimensionless, Figure 7

Appropriate stress concentration factors (SCF) for each shaft section are also included in the stress calculations. Unless otherwise specified, the keyways are assumed to comply with the standard USAS geometry which has a fillet radius of 2 percent of the shaft diameter at the corners of the keyway. The stress concentration factor is approximately 3 for a USAS keyway. If the keyway has a sharp corner at the bottom this stress concentration factor can be significantly greater.

If the shaft has stepped sections or other sudden geometric changes such as keyways or splines, then a stress concentration factor should be applied to the calculated nominal stresses. Stress concentration factors can be determined from the geometry of each shaft of keyway. Typical SCF values are approximately 1.1–2.0 for stepped shafts and 2.5–4.0 for keyways (*Stress Concentration Factors*, Peterson, Ref. 5).

The total combined stress can be calculated by combining all of the individual stress harmonics since the phasing of each forcing function is considered. The effect of combining stress components in a motor shaft throughout the variable speed range is shown in Figure 8.

In some geared systems, the interaction between the lateral and torsional vibrations can be significant. Combined torsional and lateral analyses should be considered for critical applications.

Startup

During startup, when a system accelerates through a resonance, the stresses are dependent upon the motor acceleration, system damping and the resonant frequency. The actual acceleration rate of the equipment system is dependent upon the driver starting torque, the inertias of the equipment, friction and windage loads and break–away torque. Rapid starts require more torque which also increases the dynamic torque components which may result in significant torsional stresses. The shaft stresses were measured on the motor shaft during a VFD system start as shown in Figure 9. The first torsional resonance is excited by several harmonics in sequence as the system is started.

During startup, motors may develop a strong oscillating torque because of slippage between the rotor and stator fields caused by differences in the acceleration rate of the system and the electrical frequency ramp rate. Although this is only a transient excitation, the pulsating torque can be strong enough to exceed the torsional endurance limit of the shaft. For this reason, the transient stresses must be calculated and compared to the endurance limit stress. It is not necessary that the transient stresses be less than the endurance limit stress; however, the stresses

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must be sufficiently low to allow an acceptable number of starts. If the transient stresses exceed the endurance limit, obtained from the appropriate S–N curve, then the cumulative fatigue concept is applied to the stresses in excess of the endurance limit stress to determine how many starts can be allowed for the system. These startup stresses can be accurately calculated by performing a time-domain transient analysis; however, this is normally not needed to assess the reliability of the system, since simplified analysis will show those systems which may have problems.

These excessive startup stresses tend to be more predominant in torsional systems in which the inertia of the driven equipment is significantly greater than the motor inertia. This effect is in part because of difficulty in accelerating the large inertia, resulting in more slippage and torque. Systems with a high gear ratio experience similar problems because of the large effective inertia of the driven equipment.

Torsional Stress Criteria

The acceptability of the torsional system is determined by comparison to typical engineering criteria. Common criteria used for industrial machinery (API Standards) recommended separation of the torsional critical speed and the frequency of all driving energy by a margin of 10 percent. For long term reliability, steady-state calculated torsional cyclic stresses should be compared with the allowable torsional endurance limits for the shafting material to define the safety factor. There is limited published information on the torsional endurance limits of typical shafting material and there are some differences in the interpretation of the allowable torsional stresses. EDI has had extensive experience with investigations of torsional shaft failures and has developed the following logic and procedures for evaluating torsional acceptability. The EDI evaluation is based on the following criteria.

MIL STD 167, Type III – Torsional Vibrations

MIL STD 167 (Ref. 6) defines a torsional allowable endurance stress for steel as the ultimate tensile strength divided by 25.

$$S_e = \frac{S_U}{25}$$

where: S_e = shear stress endurance limit, psi zero–peak
S_U = ultimate tensile strength, psi

The safety factor and stress concentration factor are not included in this endurance stress.

$$SF = \frac{S_e}{S_{cal} SCF}$$

where: SF = safety factor

SCF = stress concentration factor
S_{cal} = calculated torsional stress, psi zero–peak

ASME Criteria III (Ref. 7)

ASME has developed an S–N curve for low-cycle fatigue analysis for events up to one million cycles (Figure 10). The allowable shear stress at one million cycles based on this data is 6,500 psi zero–peak for carbon steel material with an ultimate tensile strength up to 80,000 psi. This particular allowable stress value is not considered to have a safety factor. Good design practice suggests that a safety factor of 2 should be used for fatigue analysis. Therefore, with this safety factor, the allowable shear stress becomes 3,250 psi zero–peak. For steels with ultimate tensile strength from 115,000 to 130,000 psi, the allowable torsional stress at one million cycles is 10,000 psi zero-peak. If a safety factor of 2 is again used, then the allowable stress limit is 5,000 psi zero–peak.

American Society of Metals Handbook (Ref. 8)

Based on fatigue tests of 4340 steel, the fatigue limit of steel with various UTS is presented in Figure 11. For a 99% survival rate, the steel could withstand a tensile endurance limit stress of 60,000 psi 0–p or equivalently by a shear stress of 30,000 psi 0–p, based upon the theory of maximum shear failure. Since yielding is undesirable, the endurance limit based upon strain (Figure 12) would be defined as:

$$S_e = \frac{\Delta\epsilon\,\mathrm{E}}{2} \quad \text{psi zero-peak for tension} = 34,000 \text{ psi 0–p (from Figure 12)}$$

Therefore the endurance limit for shear would be 17,000 psi 0–p.

Since these endurance limit stress values are based on polished test specimen, several factors must be applied to provide a reasonable endurance limit for industrial application to shafts. These factors were formulated by Marin and Shigley (Ref. 9) to compensate for surface finish, size, reliability, temperature, stress concentration, etc.

$$S_e = k_a\,k_b\,k_c\,k_d\,k_e\,\ldots\,\sigma_e$$

S_e = endurance limit psi p–p

σ_e = fatigue limit of specimen psi p–p

k_a = surface finish = 0.8 (machined)

k_b = size and shape = 0.6 (3″ diameter)

k_c = reliability (applied to graph)

k_d = temperature = 1 (for low temperature)

k_e = applied to calculated stresses

k_f = residual stress—none

k_g = interstructure (metallurgical)

k_h = environment

k_i = surface treatment

k_j = fretting

k_k = shock or vibration loading

For ordinary applications the correction factors would be:

$$k_a\, k_b\, k_c\, k_d\, k_d\, k_e = (.8)(.6)(1)(1)(1)(1) = .48$$

The allowable endurance stress in shear for AISI 4340 steel would be $(17,000)(.48) = 8,160$ psi 0–p for the 3 inch diameter specimen. This value falls below the ASME value of 10,000 psi 0–p before a safety factor is applied.

When the allowables from US MIL STD 167 and the ASME criteria are compared, reasonable agreement is obtained.

Allowable Torsional Stresses AISI 4340 Steel psi, zero–peak			
Ultimate Tensile Strength, psi	US MIL STD 167	ASME Criteria III	ASM 3" Diam
80,000	3,250	6,500	—
130,000	5,200	10,000	8,160

Thus the MIL STD 167 criteria gives conservative allowable torsional stress values compared to the ASME and ASM criteria. EDI's field experience on many systems that have been analyzed and tested has shown the MIL STD 167 to be appropriate (conservative) and is the one that EDI recommends to use. Each of these criteria should be used with a safety factor, normally a factor of 2, to compensate for other additive stresses which may exist, such as lateral vibratory stresses.

The ultimate tensile strength has a direct consequence on the torsional endurance limit of the shaft material. Based upon the US MIL STD 167, the endurance limit is a direct function of the ultimate tensile strength of the steel. For instance, a steel with 65,000 psi ultimate tensile

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strength would have 5,200 psi p–p torsional endurance limit. A steel with an 80,000 psi ultimate tensile would have 6,400 psi p–p allowable and a steel the 100,000 psi ultimate tensile strength would have 8,000 psi p–p torsional endurance limit. For systems expected to experience high dynamic torques, such as variable frequency drive motors, good quality steel with higher UTS should be used to provide a better endurance limit.

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Synchronous Motors

After the steady-state analysis has been performed, a transient analysis can be made to evaluate the startup (Ref. 10). The transient analysis refers to the conditions on startup which are continually changing because of the increasing torque and speed of the system. When a synchronous motor starts, an excitation is imposed upon the torsional system due to field slippage. As the motor increases in speed, the torsional excitation frequency decreases toward zero. During this startup, the torsional system will be excited at its resonant frequencies below 120 Hz, as shown in Figure 13. The response amplitude and shaft stresses depend upon the resonant frequencies, the average and pulsating torque when the system passes through these resonant frequencies, the damping in the system, and the compressor load torques. The analyses can be made for startup with compressors loaded or unloaded. The transient response is also affected by the starting acceleration rate of the motor. For slower motor startups, the system will stay at a resonant frequency for a longer period of time allowing stresses to be amplified. If acceleration is rapid, then passing through the resonance quickly will minimize the amplification at resonant frequencies.

Synchronous motors develop a strong oscillating torque during startup because of slippage between the rotor and stator fields. Although this is only a transient excitation, the pulsating torque can be strong enough to exceed the torsional endurance limit of the shaft. For this reason, the transient stresses must be calculated and compared to the endurance limit stress. It is not necessary that the transient stresses be less than the endurance limit stress; however, the stresses must be sufficiently low to allow an acceptable number of starts. If the transient stresses exceed the endurance limit, obtained from the appropriate S–N curve, then the cumulative fatigue concept is applied to the stresses in excess of the endurance limit stress to determine how many starts can be allowed for the system.

If the ultimate tensile strength were increased by changing the shaft material, then the allowable number startups would also increase. However, the number of allowable starts is not a direct linear function of the UTS.

Short Circuit Analysis

Short circuit analyses are performed in the time domain to determine the peak transmitted torques and shaft stresses. The analyses are performed with the motor speed adjusted so that the applied electrical frequency is coincident with the torsional resonant frequency to produce the maximum dynamic torque (conservative assumption). The motor air gap torque, as a function of time for the short circuit analyses, is usually obtained from the motor manufacturer.

Line-to-Line Short Circuits

A line-to-line short circuit is a short between two of the phase circuits while the motor is running. It produces a braking torque which has fundamental and second order frequency components. When analyzing a line-to-line short circuit, the motor should be run with an electrical frequency equal to the torsional natural frequency and also one-half the torsional natural frequency. The analysis should only include frequencies within the electrical frequency range (0–60 Hz).

Three Phase Short Circuit

The three phase short circuit would be with all three phases shorted together. It produces a braking torque at the fundamental (no second order).

Peak Torques

The peak torques experienced by the motor during a short circuit fault should be converted to stress values and compared to the material yield strength. This peak torque usually results in maximum torsional shear stresses in the shaft at the keyway under the coupling hub. For this reason, the keyway geometry becomes important in regard to stress risers. The stress concentration factor at the keyway depends upon the fillet radius at the root of the keyway according to Peterson (Ref.5). Reduction of keyway stresses is a major purpose behind the ASME code USAS B17.1, *"Keys and Keyseats"*, which defines keyway size and fillet radius.

If the yield stress is exceeded, local yielding would be expected, normally in the key or at the keyway. Under shear loading, the steel will yield at one-half of the tensile yield strength. Macro-yielding of the shaft would not be expected except for gross overloads when the ultimate strength is exceeded. The manufacturer of the equipment should review the imposed dynamic torques during startup and the peak torques of a line-to-line short circuit.

If multiple short circuit faults occur, the stresses which exceed the allowable torsional endurance stress could cause fatigue in the shafts. A cumulative fatigue analysis could be made to determine the percentage of fatigue which has been accumulated for a short circuit fault.

Cumulative Fatigue

Since stresses occurring during startup are transient, they should be evaluated on the basis of cumulative fatigue rather than compared to US MIL STD 167 which is applicable to steady-state stresses occurring within the operating speed range. Cumulative fatigue theory is used to estimate how many cycles of a certain stress level can be endured before shaft failure would occur (Figure 14). This is based upon a plot of stress versus number of cycles (S–N curve) which defines the number of cycles at a particular stress level which would result in a failure. The S–N curves used to determine the allowable number of startups are based upon full size test specimens. These S–N curves are available for most types of shafting materials (Ref. 8).

In order to determine a number of allowable starts, the number of cycles at each stress level which would occur during startup could be calculated by performing a time-domain (transient) startup analysis. Using the S–N curve, the allowed number of cycles for a particular stress level can be determined. If these cycles occurred in the transient startup, then the number of total startups can be calculated which can be made with the system before a shaft failure would be expected. Since the stress levels vary both in amplitude and frequency, a more complex calculation must be made to determine the fraction of the total fatigue which has occurred. The stress levels for each cycle are analyzed to determine the percentage of cumulative fatigue experienced during each start and then the allowable number of startups can be determined.

Definition of Terms:

1. The *endurance limit* refers to the maximum stress level which would not cause a failure to occur even for an infinite number of stress cycles.
2. The *electrical frequency* is the fundamental exciting frequency providing the driving torque to a motor. It may be different from line frequency (60 Hz) in the case of a variable frequency drive which varies the frequency from 2 to 60 Hz.
3. The *full load rated torque* is the torque required to drive the system at rated speed for full load conditions.
4. *Mass-elastic properties* refers to the mass inertias (WR^2) and the stiffnesses (K) of the motor, fan, couplings and shafts. The natural torsional frequencies of the system are dependent upon these mass-elastic properties.
5. The *torsional natural frequency* is a frequency which requires minimal energy to respond to large amplitudes. It is determined by the mass-elastic properties of the shaft system. Critical speed and resonant frequency are equivalent terms.
6. *Peak-to-peak amplitude* is the difference between the instantaneous minimum amplitude and the subsequent maximum amplitude of the complex waveform. It is sometimes referred to as the double-amplitude.
7. *Vibratory or dynamic torque* refers to the dynamic (modulating) component of torque in the coupling or in the shaft which is a result of the response of the system to the input and load torques of the motor and fan.
8. *Zero–peak amplitude* is the difference between the average or equilibrium amplitude and the instantaneous maximum or minimum value of the complex waveform. It is sometimes referred to as the half-amplitude.

References:

1. Harris, C.M., and Crede, C.E., *Shock and Vibration Handbook*, Chapter 38, 2nd Ed., McGraw-Hill Book Co., 1976.
2. Nestorides, E.J., *A Handbook on Torsional Vibration*, Internal Combustion Engine Research Association, 1958, pp. 84–88.
3. Ker Wilson, W., *Practical Solution of Torsional Vibration*, British Internal Combustion Engine Research Association, 1958, pp. 84–88.
4. Wachel, J. C. et al., *Rotordynamics of Machinery*, 1986.
5. Peterson, R.E., *Stress Concentration Factors*, John Wiley & Sons, New York, 1974.

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6. MIL STD 167–1 (SHIPS) May 1974, Mechanical Vibrations of Shipboard Equipment (Reciprocating Machinery and Propulsion System and Shafting) Type III, IV and V.

7. Criteria of Section III of the ASME Boiler and Pressure Vessel Code for Nuclear Vessels.

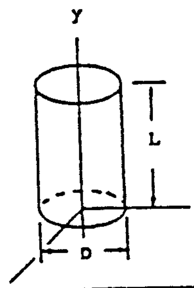
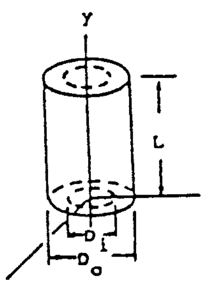
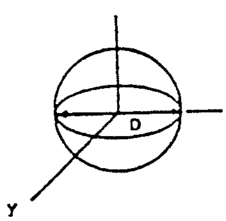
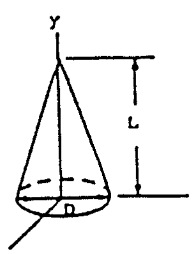
8. American Society of Metals, *Atlas of Fatigue Curves*, Editor H.E. Boyer, ASM, 1986.

9. Shigley, J. E., *Mechanical Engineering Design*, 4th ed., McGraw Hill.

10. Szenasi, F. R., and Von Nimitz, W. W., "Analyses of Synchronous Motor Trains", 7^{th} Turbomachinery Symposium, Gas Turbine Laboratories, Texas A&M University, College Station, Texas, 1978.

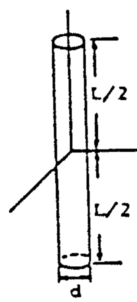
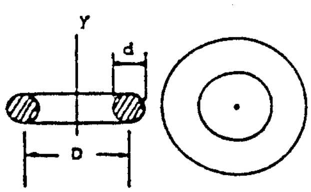
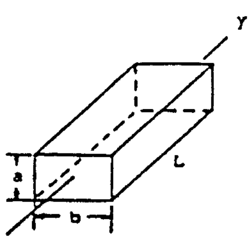
11. USAS B17.1 - 1967 (1973), *Keys and Keyseats*, American Society of Mechanical Engineers.

FIGURE 1
POLAR MOMENTS OF INERTIA
FOR SELECTED CONFIGURATIONS

	Description	Radius of Gyration	Mass	Polar Moment of Inertia
	Solid Circular Cylinder	$\frac{D}{2\sqrt{2}}$	$\frac{\gamma\pi D^2 L}{4g}$	$\frac{\gamma\pi L D^4}{32g}$
	Hollow Circular Cylinder	$\sqrt{\frac{D_o^2 + D_i^2}{8}}$	$\frac{\gamma\pi L[D_o^2 - D_i^2]}{4g}$	$\frac{\gamma\pi L[D_o^4 - D_i^4]}{32g}$
	Sphere	$\frac{D}{\sqrt{10}}$	$\frac{\gamma\pi D^3}{6g}$	$\frac{\gamma\pi D^5}{60g}$
	Circular Cone	$\sqrt{\frac{3D^2}{40}}$	$\frac{\pi\gamma D^2 L}{12g}$	$\frac{\gamma\pi L D^4}{160g}$

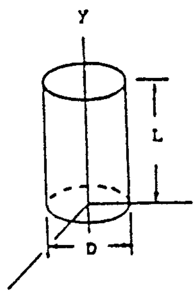
γ = Weight Density
g = Acceleration of Gravity

FIGURE 1 (cont'd)
POLAR MOMENTS OF INERTIA
FOR SELECTED CONFIGURATIONS

	Description	Radius of Gyration	Mass	Polar Moment of Inertia
	Slender Rod	$\sqrt{\frac{L^2}{12}}$	$\frac{\gamma\pi d^2 L}{4g}$	$\frac{\gamma\pi d^2 L^3}{48g}$
	Torus	$\sqrt{\frac{D^2}{4} + \frac{3d^2}{16}}$	$\frac{\gamma\pi^2 D d^2}{4g}$	$\frac{\gamma\pi^2 D d^2}{4g}\left[\frac{D^2}{4} + \frac{3d^2}{16}\right]$
	Rectangular Prism	$\sqrt{\frac{a^2 + b^2}{12}}$	$\frac{\gamma abL}{g}$	$\frac{\gamma abL}{g}\left[\frac{a^2 + b^2}{12}\right]$

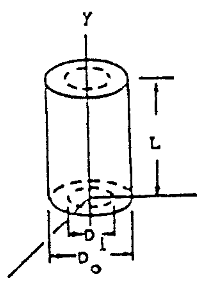
γ = Weight Density
g = Acceleration of Gravity

FIGURE 2
TORSIONAL STIFFNESS
OF SELECTED CROSS-SECTIONS



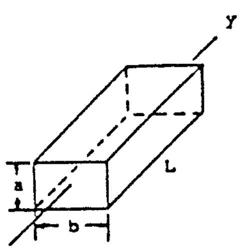
Solid Circular Cylinder

$$K = \frac{\pi G D^4}{32L}$$



Hollow Circular Cylinder

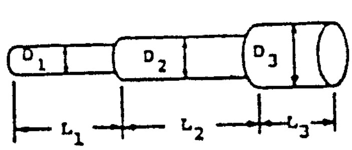
$$K = \frac{\pi G [D_o^4 - D_i^4]}{32L}$$



Rectangular Prism

$$K = \frac{C_1 G a b^3}{L}$$

a/b	1	2	3	5	10
C_1	0.141	0.229	0.263	0.291	0.312



Stepped Shaft

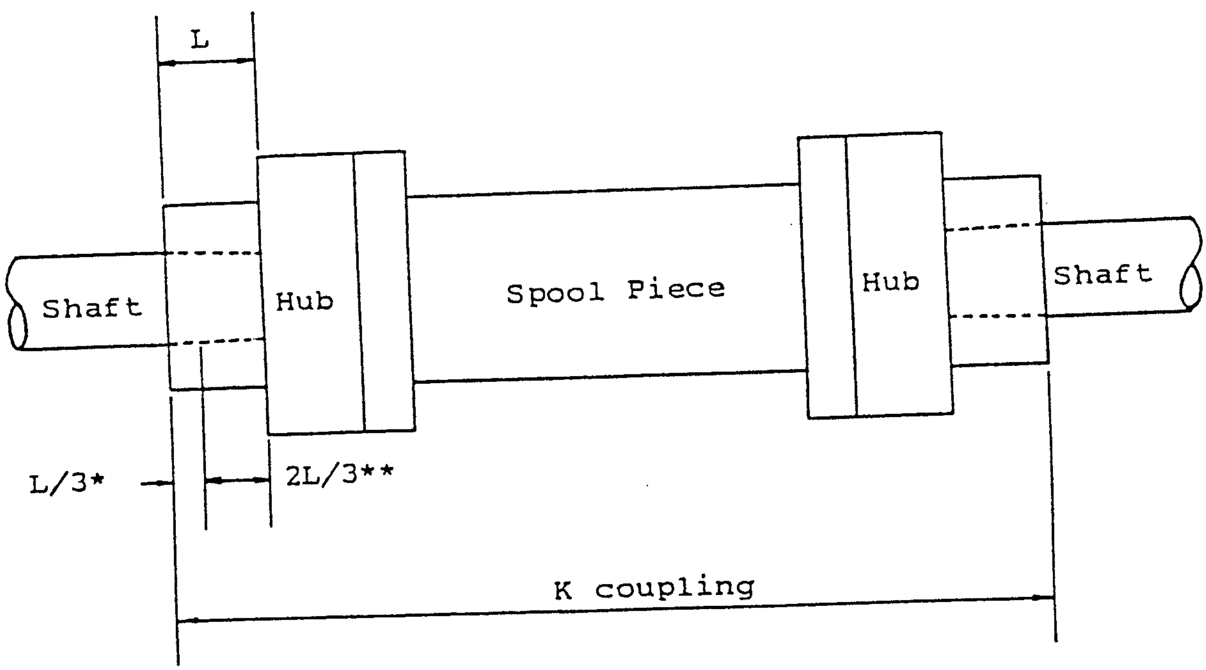
$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$$

$$= \frac{32}{\pi G}\left[\frac{L_1}{D_1^4} + \frac{L_2}{D_2^4} + \frac{L_3}{D_3^4}\right]$$

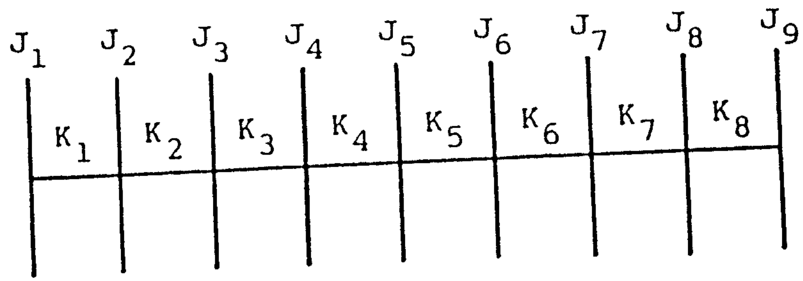
G = Shear Modulus
K = Torsional Stiffness

FIGURE 3

SHAFT PENETRATION ASSUMPTION



* Shaft twists freely for L/3
** No slippage for 2L/3



$$
\begin{bmatrix}
J_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & J_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & J_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & J_4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & J_5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & J_6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & J_7 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & J_8 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_9
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \\ \ddot{\theta}_4 \\ \ddot{\theta}_5 \\ \ddot{\theta}_6 \\ \ddot{\theta}_7 \\ \ddot{\theta}_8 \\ \ddot{\theta}_9
\end{bmatrix}
= -
\begin{bmatrix}
K_1 & -K_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-K_1 & K_1 + K_2 & -K_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -K_2 & K_2 + K_3 & -K_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -K_3 & K_3 + K_4 & -K_4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -K_4 & K_4 + K_5 & -K_5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -K_5 & K_5 + K_6 & -K_6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -K_6 & K_6 + K_7 & -K_7 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -K_7 & K_7 + K_8 & -K_8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -K_8 & K_8
\end{bmatrix}
\begin{bmatrix}
\theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \\ \theta_9
\end{bmatrix}
$$

FIGURE 4

TORSIONAL ANALYSIS

EIGENVALUE MATRIX EQUATION

ROTOR-SHAFT SYSTEMS
(RIGID ROTOR AND MASSLESS SHAFT)

k_t = TORSIONAL STIFFNESS OF SHAFT, LB-IN./RAD

I = MASS MOMENT OF INERTIA OF ROTOR, LB-IN.-SEC^2

ω_n = ANGULAR NATURAL FREQUENCY, RAD/SEC

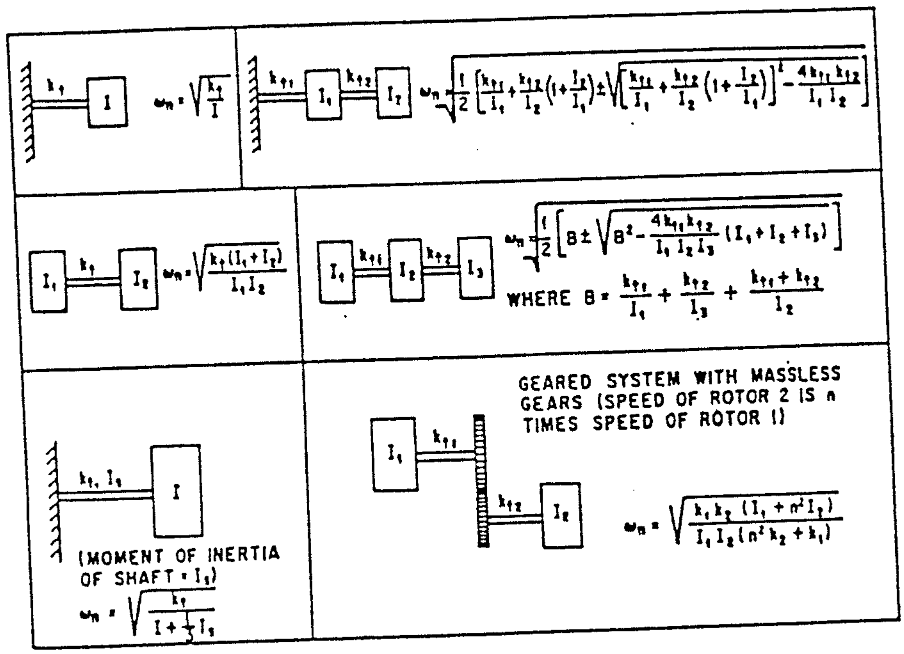


FIGURE 5

FREQUENCY CALCULATIONS
FOR SIMPLE SYSTEMS

VFD MOTOR AND FORCED DRAFT FAN
1500 HP MOTOR - 1200 RPM
COMBINED STRESSES

INTERFERENCE DIAGRAM OF TORSIONAL RESONANCES

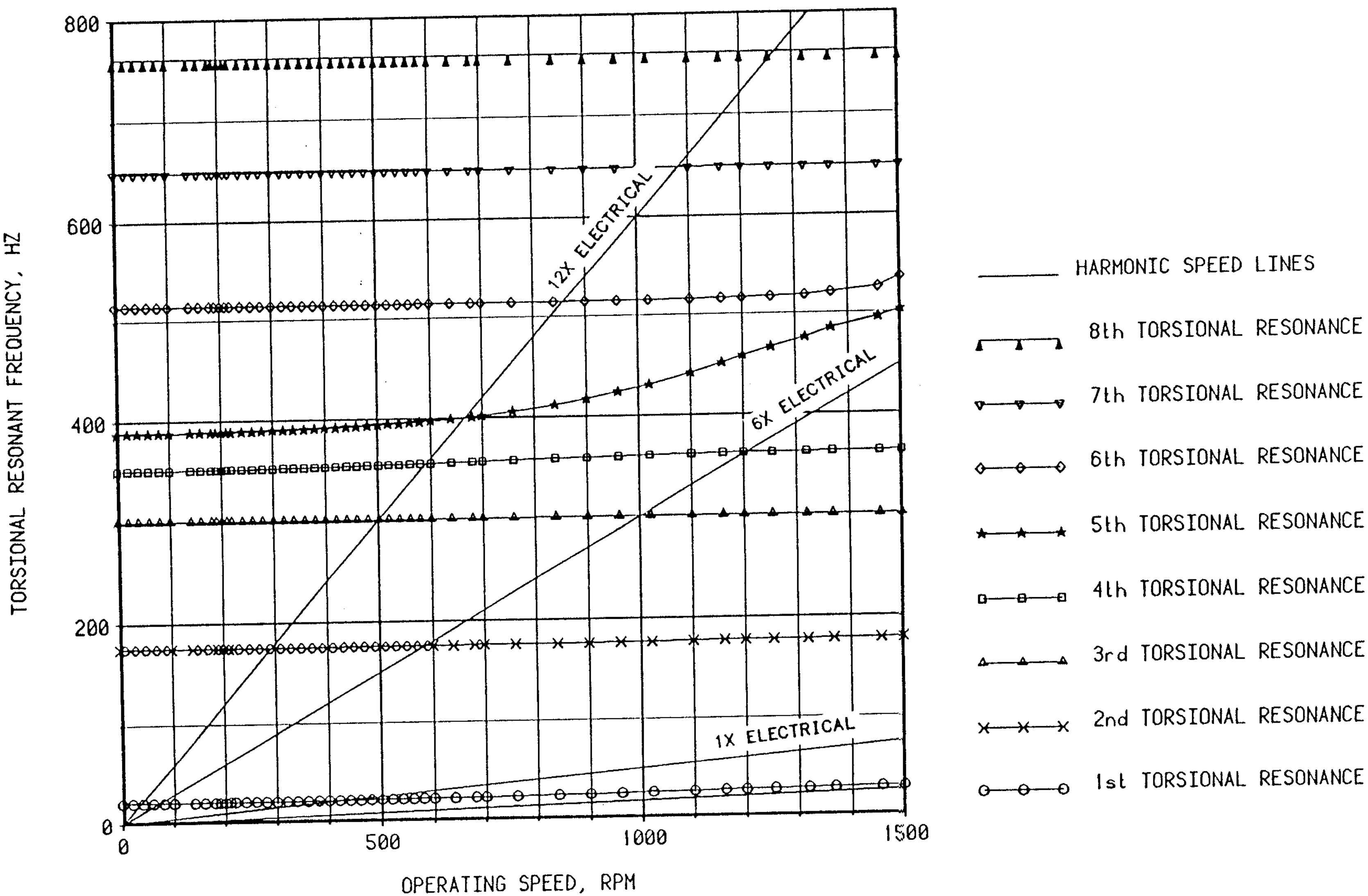
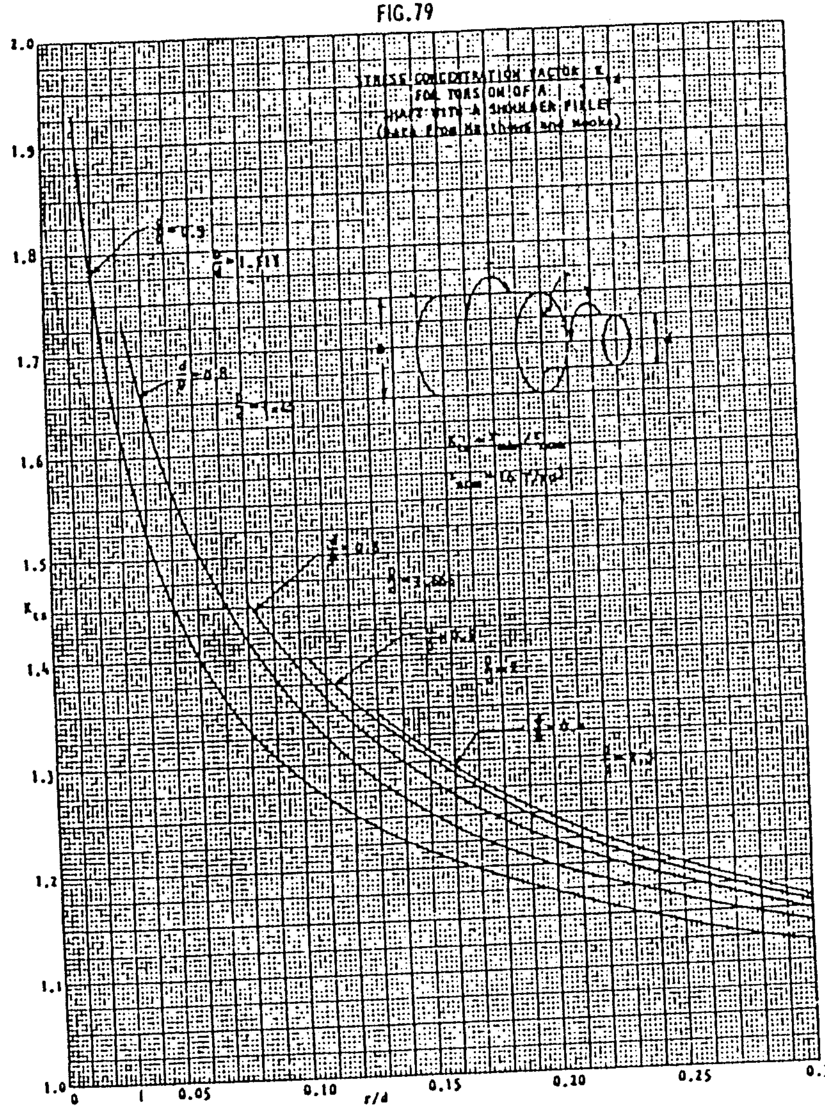


Figure 6

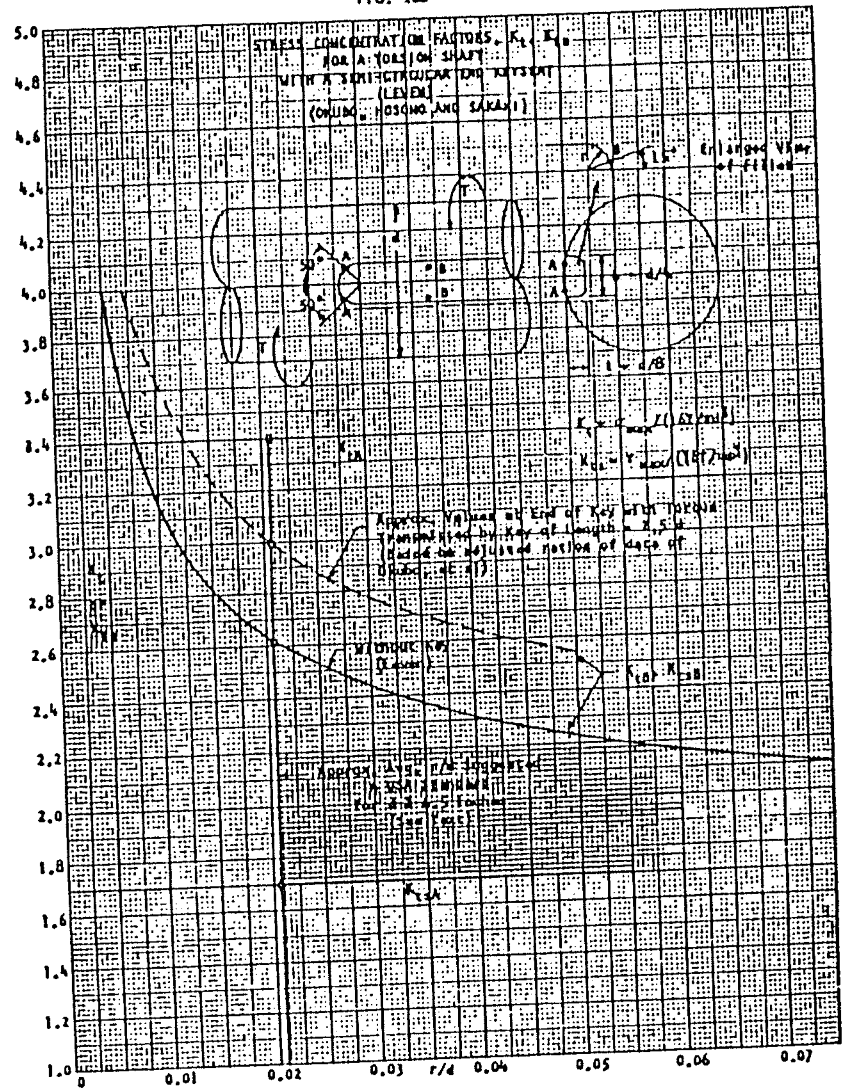
Engineering Dynamics Incorporated - San Antonio, Texas

FIG.79



Stepped Shaft

FIG. 183



Keyways

Figure 7

Stress Concentration Factors

(Ref. book of same title; R. E. Peterson
John Wiley & Sons, New York, 1974.)

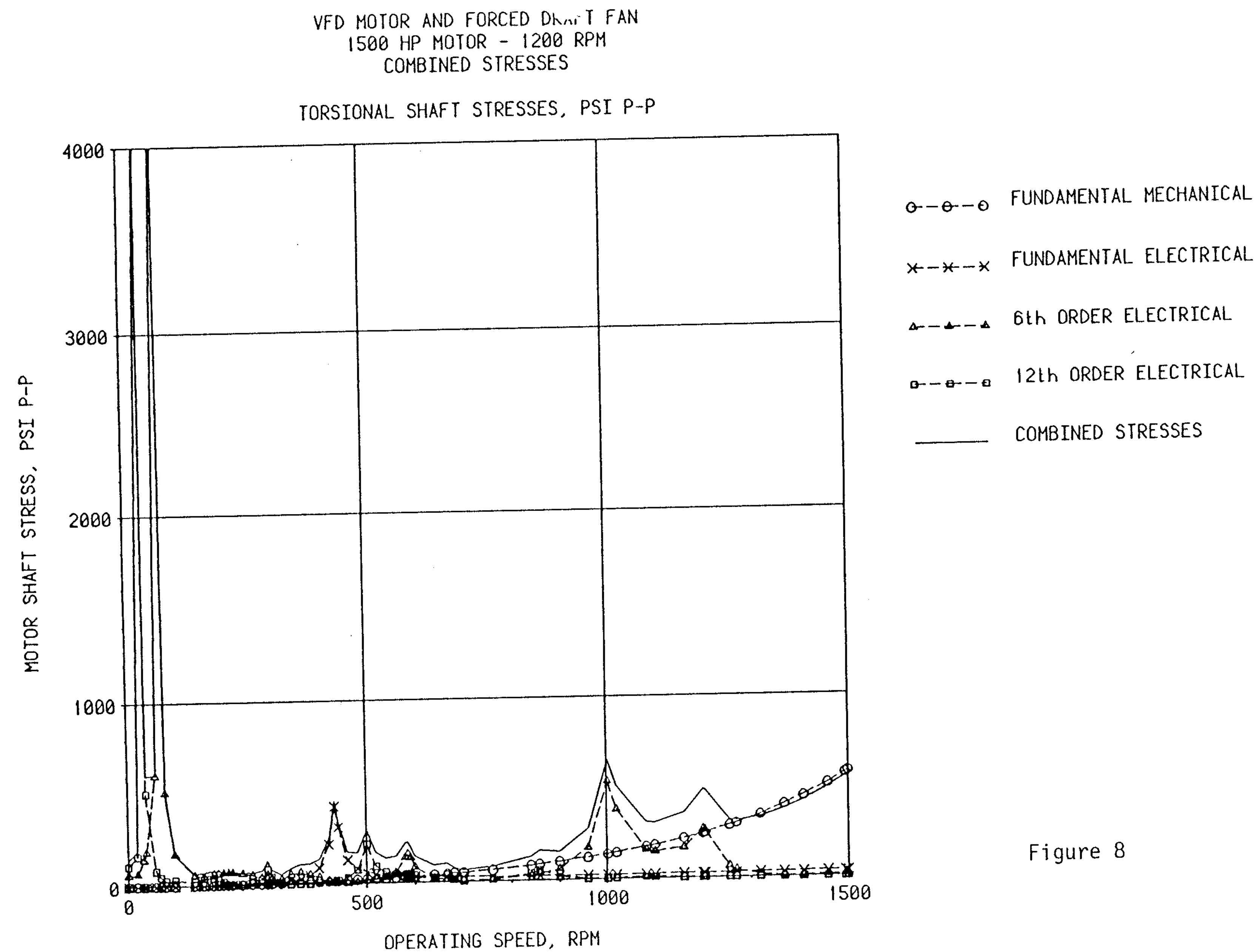


Figure 8

Engineering Dynamics Incorporated - San Antonio, Texas

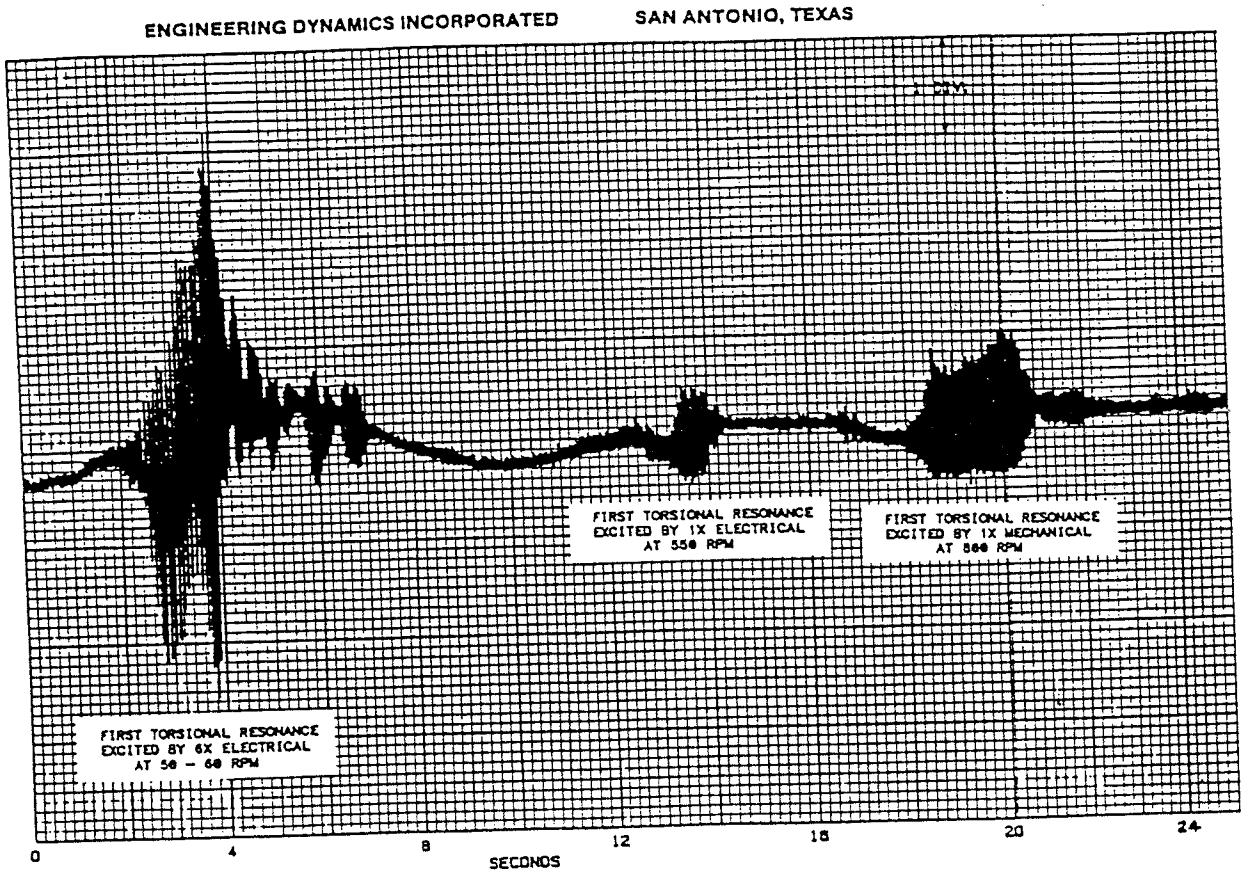


Figure 9

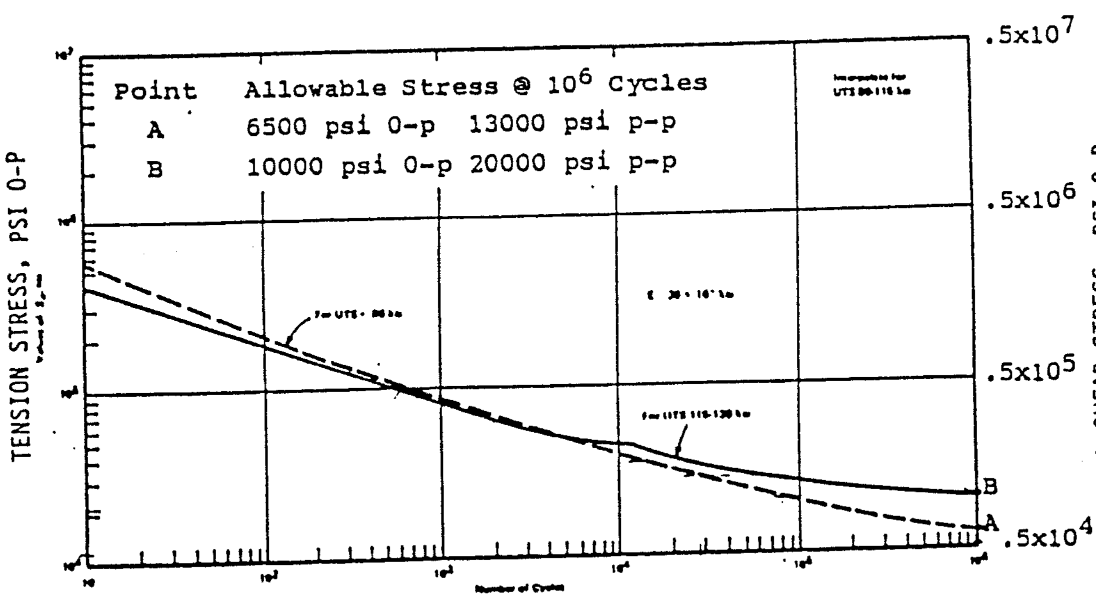


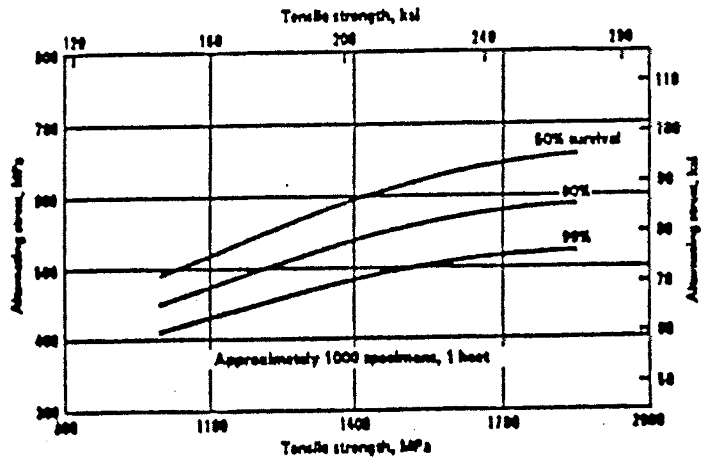
FIG. 5-110.1 DESIGN FATIGUE CURVES FOR CARBON, LOW-ALLOY, SERIES 4XX, HIGH-ALLOY STEELS AND HIGH TENSILE STEELS FOR TEMPERATURES NOT EXCEEDING 700 F

Criteria of Section III of the ASME Boiler and Pressure Vessel Code for Nuclear Vessels

Figure 10

Figure 11

4-15. 4340 Steel: Scatter of Fatigue Limit Data



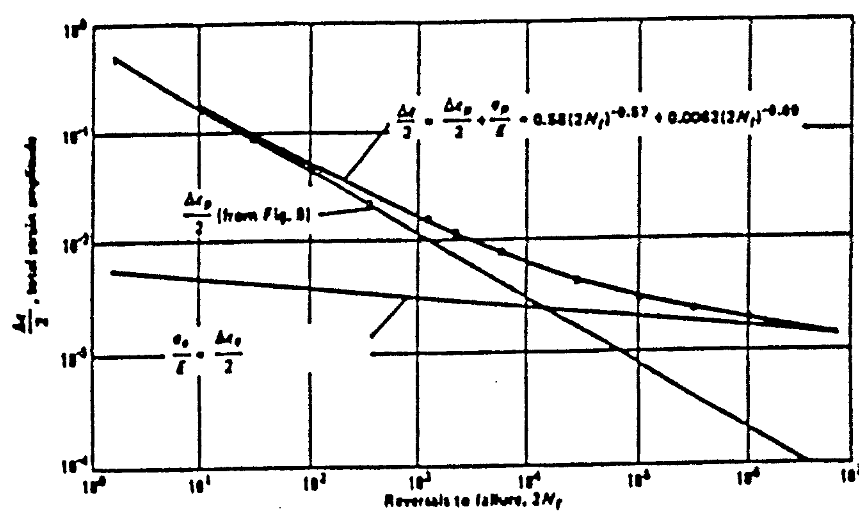
Interrelationships of alternating stress, tensile strength and expected percent survival for heat treated 4340 steel.

These data show survival after 10 million cycles of AISI-SAE 4340 steel with tensile strengths of 995, 1320, and 1840 MPa (144, 191, and 267 ksi). Rotating-beam fatigue specimens tested at 10 000 to 11 000 rpm. Coefficients of variation range from 0.17 to 0.20. From these data it is evident that scatter increases as strength level is increased.

Source: Metals Handbook, 9th Edition, Volume 1, Properties and Selection: Irons and Steels, American Society for Metals

Figure 12

4-17. 4340 Steel: Total Strain vs Fatigue Life



Typical data for total strain versus fatigue life for annealed 4340 steel.

Source: Metals Handbook, 9th Edition, Volume 1, Properties and Selection: Irons and Steels, American Society for Metals.

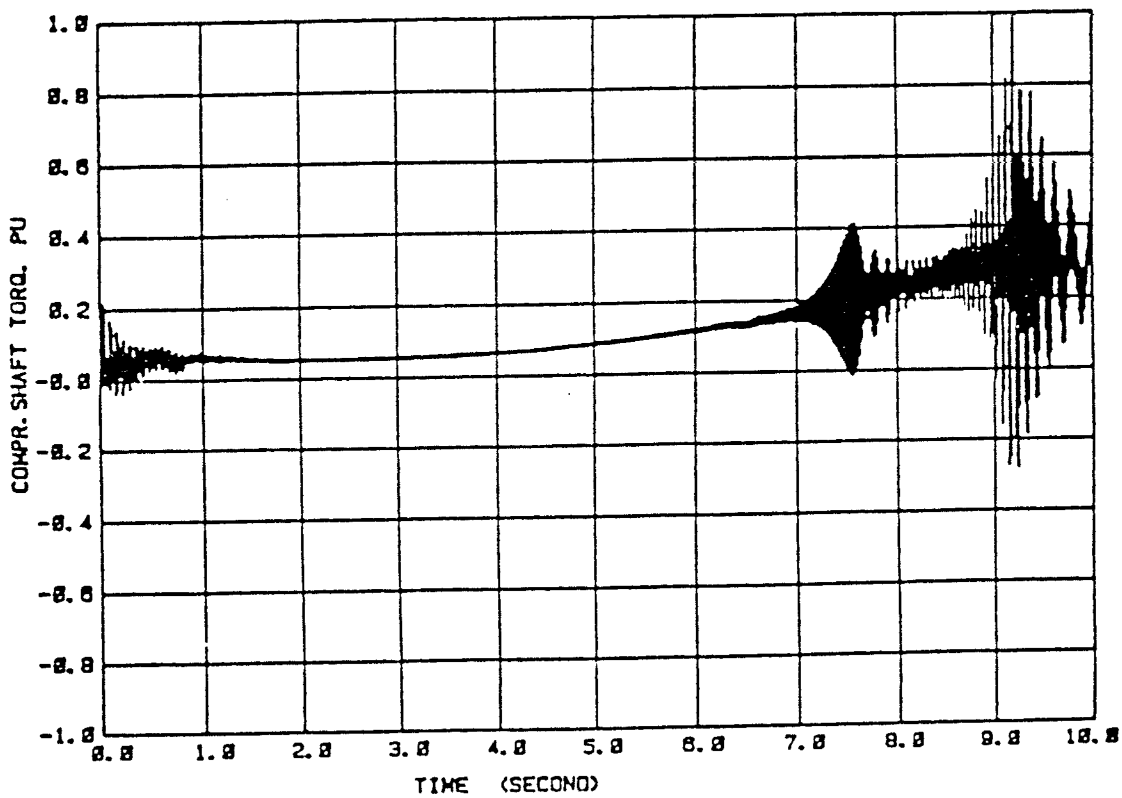


Figure 13

Transient Torsional Startup (Ref. 5)

σ_i = STRESS AT i^{th} CYCLE

σ_e = ENDURANCE STRESS

N_i = NO. OF CYCLES TO FAILURE AT σ_i

n_i = ACTUAL NO. OF CYCLES AT σ_i

S = NO. OF CYCLES TO SYNCHRONOUS SPEED

$$q = \sum_{i=1}^{s} \frac{n_i}{N_i}$$

$\frac{1}{q}$ = ALLOWABLE NUMBER OF START UPS

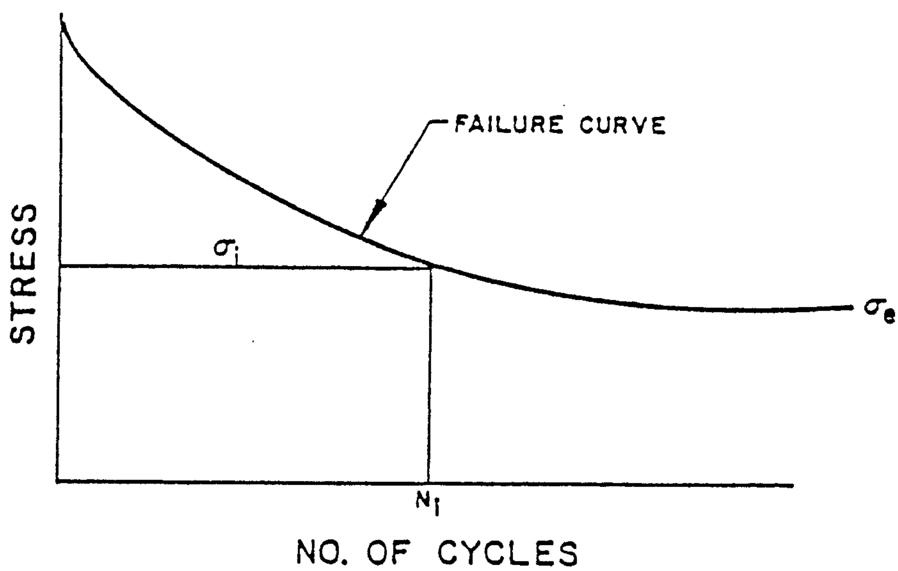


Figure 14
Cumulative Fatigue