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DISPLACEMENT METHOD FOR DETERMINING ACCEPTABLE PIPING VIBRATION AMPLITUDES

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ABSTRACT

Methods are presented in the ANSI/ASME Operation & Maintenance Standards/Guides Part-3 1991, "Preoperational and Initial Startup Testing of Nuclear Power Plant Piping Systems" (OM-3) for determining if the vibration-induced stresses in a vibrating pipe span are acceptable. The displacement and velocity methods are presented in OM-3. These methods are applicable for piping spans vibrating at resonance in lateral bending modes. The background and theory necessary for understanding the lateral bending vibration modes of typical piping systems and the procedures for evaluating stresses are discussed.

The stress factors for characteristic spans given in OM-3 do not cover the entire range of configurations encountered in actual piping systems. In addition, data for calculating the lateral bending natural frequencies of piping spans and bends are not included in the OM-3 guidelines. Frequency factors, deflection stress factors and velocity stress factors for calculating the mechanical natural frequencies and the vibration-induced stresses of most common piping spans have been published (Wachel et al., 1990). This information was used to develop a nomogram which can be used to calculate the natural frequency, the vibration-induced stress, and the allowable vibration amplitude of most piping spans.

The information presented herein on piping natural frequencies and vibration-induced stresses should allow the vibration analyst to evaluate piping systems under the OM-3 Vibration Monitoring Group 2 (VMG-2) category in less time and with increased confidence in the results.

INTRODUCTION

Most piping will experience some amount of steady-state vibrations as a result of the excitation sources in the system. Piping vibration excitation mechanisms include pressure pulsations in the fluid being transported by the piping, and vibrations mechanically transmitted by attached or adjacent equipment, as discussed in Part 3, Appendix E of OM-3. Figure 1 summarizes the most common excitation mechanisms for piping vibration.

The vibratory response of the piping system is a function of (1) the piping mechanical natural frequencies which are dependent on the pipe diameter, span length, effectiveness of the pipe supports, restraints, guides, anchors, and snubbers, (2) the acoustical resonant frequencies of the piping system which are influenced by the acoustical properties of the fluid, (3) the piping system operational conditions, such as pressure, temperature, flow, multiple unit operation and the kind of operational transient (startup, shutdown, etc.), (4) the flow-induced pulsations which are dependent upon the flow velocity, Strouhal number, main and branch pipe diameters, and the presence of obstructions such as orifices, valves, condenser tubes, etc, and (5) pulsations generated from reciprocating machinery.

In the design stage, it is difficult to calculate the pulsations that will exist in the piping due to flow-excited mechanisms. The acoustical natural frequencies of piping systems and the potential for the flow excitation of the acoustical natural frequencies can be calculated; however, the amplitudes of the flow induced pulsation and the severity of the problem are difficult to accurately predict.

Pulsation characteristics from reciprocating machinery, such as positive displacement charging pumps, can be calculated by using digital computer programs or analog simulators which model the acoustical properties of the pump and the piping system. Many piping vibration-induced failures have occurred in charging pump systems (Olson, 1985). These could have been avoided by taking advantage of the improvements in the state-of-the-art in the design of proper pulsation control in reciprocating pump systems (Wachel et al., 1993). For example, the latest revision (scheduled to be published in 1995) of the American Petroleum Institute Code for Reciprocating Pumps API 674 will require that an acoustical analysis of the pump and piping system be made to develop adequate pulsation and vibration control for critical systems.

Steady state piping vibration in nuclear power plants is considered in the design stage by using the experience with previous systems and good engineering practices in the layout and supporting of the piping systems to minimize vibration problems. The piping vibrations must be qualified to ensure that fatigue failure will not result. The predominant way of accounting for these vibrations is to monitor the piping vibration levels during startup and actual operation. Because of the immense amount of piping typically found in a power plant, simplified qualification techniques are needed to keep the task of addressing the vibrations within manageable limits.

Most dynamic fatigue failures in piping systems are caused by individual piping spans vibrating at resonance. The stress in a piping span which is vibrating at resonance is directly proportional to the maximum vibration amplitude (displacement, velocity, or acceleration) in the span. In order to determine if measured vibration amplitudes of piping systems are acceptable, the dynamic stresses caused by the vibrations should be compared to the applicable endurance limit for the piping material.

SIMPLIFIED METHOD FOR QUALIFYING PIPING SYSTEMS

In OM-3, the maximum allowable vibration amplitudes for a piping system are evaluated in Vibration Monitoring Group 2 (VMG-2) by dividing the piping system into characteristic or analogous beam segments. It is assumed that piping vibration measurements are made on the characteristic piping span while the span is vibrating at its mechanical natural frequency and that the measured mode shape exactly matches the theoretical mode shape of the selected characteristic beam configuration. The simplified methods in OM-3 for calculating the vibration-induced stresses and the acceptable vibration amplitudes are based on the mode shape of the analogous piping characteristic span. The acceptance criteria are further based on limiting the peak vibratory stresses to a value less than or equal to the stress endurance limit for the piping material.

OM-3 (1991) uses the stress amplitude at 10^6 cycles from Figures I-9.1, I-9.2.1 and I-9.2.2 of the Boiler and Pressure Vessel Code to develop endurance limit stresses. The endurance limit for carbon steels at 10^7 cycles is obtained by reducing the value at 10^6 cycles by 0.8. To obtain an endurance limit at 10^{11} cycles, the value at 10^7 cycles is divided by 1.3. This essentially reduces the stress amplitude at 10^6 cycles by a factor of 1.625.

The measured vibrations on a piping span during a test reflect the conditions at the time of the test; however, experience has shown that an additional factor may be needed to compensate for the uncertainty in the testing conditions which may not have caused vibrations which are the highest that will be experienced.

The development of the vibration-induced stress in simple piping characteristic spans is shown below.

STRESS AS A FUNCTION OF THE VIBRATION AMPLITUDE

The dynamic stress in a vibrating pipe is related to the dynamic bending moment.

$$S = \frac{MD}{2I} \tag{1}$$

where:

S = stress, psi
D = pipe diameter, in
I = pipe moment of inertia, in^4
M = moment, in-lb

The moment is related to the piping vibration mode shape.

$$M = -EI\frac{d^2y}{dx^2} \tag{2}$$

where:

E = elastic modulus, psi

$\frac{d^2y}{dx^2}$ = second derivative of vibration mode shape

If equation (2) is substituted in equation (1), we obtain:

$$S = \frac{-ED}{2}\frac{d^2y}{dx^2} \tag{3}$$

Thus, it can be seen that the dynamic stress is a function of the material, the diameter, and the second derivative of the vibration mode shape. The vibration mode shape depends upon the piping configuration and the end conditions.

To illustrate, let us calculate the vibration-induced stress in a simply supported beam. It can be shown that the vibrating mode shape for a simply supported beam for the first mode is a sine wave. Thus,

$$y = Y_0 \sin\frac{\pi x}{\ell} \tag{4}$$

$$\frac{d^2 y}{dx^2} = -Y_0 \frac{\pi^2}{\ell^2} \sin\frac{\pi x}{\ell} \tag{5}$$

Y_0 = maximum vibration amplitude, in
ℓ = span length, in

The maximum occurs at midspan $\left(x = \frac{\ell}{2}\right)$ (6)

$$\left(\frac{d^2 y}{dx^2}\right)_{max} = -Y_0 \frac{\pi^2}{\ell^2} \sin\frac{\pi}{2} \tag{7}$$

$$= -Y_0 \frac{\pi^2}{\ell^2} \tag{8}$$

Substituting into the stress equation:

$$S = -\frac{ED}{2}\left(-Y_0 \frac{\pi^2}{\ell^2}\right) \tag{9}$$

Thus, for any pipe span configuration, it is feasible to calculate the stress which results from a given vibration amplitude providing the end conditions and vibratory mode shape are known.

The general equation relating maximum unintensified stress in a pipe to the maximum vibration amplitude along the span can be obtained by dividing equation (3) by the maximum measured vibration amplitude. Note that the maximum stress and maximum deflection are generally not at the same point:

$$\frac{S}{y} = \frac{ED}{2} \frac{|y''|}{|y|} \tag{10}$$

where:

S = stress at maximum stress point, psi

$|y''| = \frac{d^2 y}{dx^2}$ evaluated at maximum stress point

$|y|$ = deflection, inches, at maximum deflection point

Based on the characteristic mode shapes of uniform straight beams as presented by Blevins (1979) it can be shown that:

$$\frac{S}{y} = \frac{ED}{2} \frac{\lambda}{\ell^2} \frac{\left|\tilde{y}_n''\right|_{max}}{\left|\tilde{y}_n\right|_{max}} \tag{11}$$

where:

$\tilde{y}_n''$ = second derivative of characteristic mode shape for mode n
$\tilde{y}_n$ = characteristic mode shape for mode n
ℓ = span length, in
λ = frequency factor

Tables of $\tilde{y}_n''$ and $\tilde{y}_n$ are given for the first five modes for straight beams. If we define a deflection stress factor:

$$K_d = \frac{E\lambda}{2} \frac{\left|\tilde{y}_n''\right|_{max}}{\left|\tilde{y}_n\right|_{max}} \tag{12}$$

where:

K_d = Deflection Stress Factor for Mode n

In OM-3 analyses, an acceptable vibration amplitude of the piping is determined by breaking the piping system into a series of characteristic lengths and determining the stresses of the individual resonant spans. The allowable vibration can be expressed in terms of the vibration amplitude required to cause 10,000 psi stress as given below:

$$\delta_{allow} = \frac{(S_{el})(\delta_n)}{(10000)(C_2 K_2)(\alpha)} \tag{13}$$

where:

δ_{allow} = Allowable zero to peak vibration deflection limit, inches.

$S_{el} = S_A$, where S_A is the alternating stress at 10^6 cycles from Fig. I-9.1, or at 10^{11} cycles from Fig. I-9.2.2 of the BPV Code. The user shall consider the influence of temperature on the modulus of elasticity.

δ_n = value of deflection from OM-3 Fig. 1, p 29.

C_2 = Secondary stress index as defined by the BPV Code.
K_2 = Local stress index as defined by the BPV Code.

α = Allowable stress reduction factor: 1.625 for materials covered by Fig I-9.1; or 1.0 for materials covered by Fig. 1-9.2.1- or I-9.2.2 of the BPV Code.

Figure 1 in OM-3 is a nomogram which gives the nominal vibrational deflection values. The K value is given for the Characteristic Span Models in Figures 2 - 9 of OM-3. The equation for δ_n is given below:

$$\delta_n = K\frac{L^2}{D} \quad (14)$$

The relationship between the K factor given in the OM-3 Code and the deflection stress (K_d) factors by Wachel et al., (1990) is given below:

$$K = \frac{10}{K_d} \quad (15)$$

The vibration-induced stresses can be calculated by the following equation:

$$S = K_d\frac{(D)(C_2K_2)y}{L^2} \quad (16)$$

where:

S = Dynamic stress (psi),

K_d = Deflection stress factor

y = Vibration (mils), maximum amplitude measured between nodes (normally supports),

D = Outside diameter (inches),

L = Span length (ft),

The equations outlined above show the basic theory that was used to develop the methods given in OM-3 for VMG-2 analyses to determine if the measured vibration amplitudes are acceptable on the basis of vibration-induced stresses. The OM-3 guidelines allow the analyst to use the displacement or velocity method to make the comparison of vibration-induced stresses to the conservative endurance stress limits defined in the guidelines. The author believes that the deflection (displacement) method is easier to use and is less prone to inaccuracies. The stress correction factors are discussed in OM-3; however, additional stress correction factors must be considered when using the velocity method as discussed by Wachel et al., (1990). The additional considerations include concentrated weights, off-resonant excitation frequencies, and contents and insulation. The displacement method can be used to develop conservative allowable vibration amplitudes even though the natural frequency and mode shape do not exactly match the resonant frequency condition. However, it is very critical that the piping span be exactly on resonance if the velocity method is used. There are many difficulties in properly applying the velocity method and this can lead to non-conservative errors. Therefore, the author recommends that the displacement method be used.

The author and his associates have generated frequency factors, deflection stress factors and velocity stress factors which can be used to calculate the acceptable piping vibration amplitudes for most piping configurations. The piping spans were analyzed using a finite element code which included piping elbows. The various factors were calculated as a function of the aspect ratios of the leg lengths of the L-bend, Z-bend, U-bend, and the 3-dimensional bend. A summary of frequency factors, deflection stress factors, and velocity stress factors for straight piping spans and for equal leg piping bends is given in Figure 2.

Since one of the assumptions in VMG-2 is that the piping span is vibrating at its mechanical natural frequency, weight correction factors were derived for concentrated weights (valves, flanges, etc.) located within the piping span. The weight correction factors for natural frequency calculations are given in Figure 3. These factors can be used to calculate the mechanical natural frequency of piping spans with concentrated weights at various locations. A comparison of the measured and calculated resonant frequency is required to determine if the simple characteristic beam analogy is appropriate. Concentrated weights affect the vibration-induced stresses. Figure 4 gives the correction factors for the calculated stresses for the displacement and velocity methods. Note that the stress correction factor in Figure 4 is the inverse of the C_1 factor given in Figure 10 of OM-3.

In order to make the procedures given in OM-3 for determining the vibration-induced stresses easier to use, a nomogram was developed which allows the analyst to calculate the natural frequency, stress per mil and the allowable vibration amplitude (Figure 5).

NATURAL FREQUENCY AND STRESS NOMOGRAM

The frequency factors and deflection stress factors given by Wachel et al., (1990) for straight piping spans and the piping bends (L-bend, U-bend, Z-bend, and 3-dimensional bend) were used along with the information in OM-3 to develop a nomogram which can be used to determine the allowable vibration amplitudes in compliance with the endurance limits given in OM-3.

The basis of the nomogram is listed below:

1. The piping span is vibrating on resonance at its first natural frequency. The first natural frequency of piping spans with bends is assumed to correspond to the out-of-plane mode.

2. The vibration mode shape is that associated with the first natural frequency.

3. The maximum vibratory stress occurs at the location of the theoretical maximum moment.

4. The piping span is constant diameter and the radius of gyration is approximately 0.34 times the outside diameter.

5. There are no concentrated weights within the span and the weight of the fluid and insulation do not affect the vibration mode shape.

The following equations are solved by the nomogram. Each of these will be discussed below:

$$f = 75.8\lambda\left(\frac{D}{L^2}\right) \tag{17}$$

$$\frac{S}{y} = K_d\left(\frac{D}{L^2}\right) \tag{18}$$

$$y_{all} = \frac{3000}{\frac{S}{y}} \tag{19}$$

USE OF NOMOGRAM

Determine the Piping Span Natural Frequency

To determine the first natural frequency of the piping span; first, connect the piping total span length (L) to the outside diameter (D) of the pipe. Note that the outside diameter of the nominal pipe sizes has been noted on the outside diameter scale. The length L is the sum of the distances from the boundaries to the intersection of the leg lengths (A + B + C). (When the piping spans were analyzed, the piping elbows were modeled; however, the results have been plotted in terms of the sums of the lengths to the intersections as opposed to the developed lengths.) The line between the total span length and the outside diameter locates a point R on the reference line.

The second step to determine the mechanical natural frequency of the span is to connect the point on the reference line to the frequency factor λ for the characteristic piping span or piping bend. Several of the typical piping spans have been noted on the λ scale; however, if the span is one of the piping spans which has bends in it, the frequency factor can be determined from the curves on the four graphs included on the nomogram. The graphs give the frequency factors and the deflection stress factors for the L-bend, Z-bend, U-bend, and the 3-dimensional bend. The factors are plotted as a function of the leg length aspect ratios (B/A and C/A). Two curves are shown for the C/A ratios, the 0.5 and the 1.0 ratios. If the aspect ratio is within this range, the intermediate ratios can be interpolated.

Locate the frequency factor on the λ scale and connect it with the point on the reference line. The piping span natural frequency is located on the f scale which is located adjacent to the right of the reference line.

Calculate the Stress per Mil of Vibration

To determine the stress per mil that is caused by the piping span vibrating at its first natural frequency, the point on the reference line is connected to the deflection stress factor for the characteristic piping straight span or piping bend. If the appropriate deflection stress factor is marked on the K_d scale, it can be connected with the point on the reference line. If the piping span has piping bends, the appropriate deflection stress factor can be determined from the four graphs provided. Again, the aspect ratios are used to determine the deflection stress factors.

To determine the stress per mil of vibration for the piping characteristic span, connect the deflection stress factor to the point on the reference line. The stress per mil scale (S/y) is adjacent to the right of the frequency scale. The stress per mil determined from the (S/y) scale is the nominal stress and does not include the C_2K_2 intensification factors. If it is desired to calculate the measured vibration-induced stresses, the value of stress per mil from the (S/y) scale must be multiplied by the C_2K_2 and the measured maximum vibration at the antinode of the piping span.

$$S_{max} = \left(\frac{S}{y}\right) y_{meas} C_2 K_2 \tag{20}$$

The safety factor can then be calculated by comparison of the calculated vibration-induced stress with the appropriate endurance limit given in OM-3 guidelines.

Allowable Vibration Amplitudes

The allowable vibration amplitudes can be calculated using the allowable endurance limits given in the OM-3 guidelines. For carbon steel piping with an ultimate tensile strength less than 80,000 psi, the allowable is 7690 psi. For the higher strength low alloy steel of 115,000 to 130,000 psi ultimate tensile strength, the allowable is equal to 12,300 psi. For stainless steels, the lowest allowable endurance limit is equal to 13,600 psi.

The value of stress per mil (S/y) read from the scale can be multiplied by the C_2K_2 value and the measured vibration in mils (y_{meas}) to determine the vibration-induced stress. If the calculated stress is less than the allowable given in the OM-3 guidelines, the measured vibrations are acceptable and the piping span meets the acceptance criteria given in OM-3.

The BPV criteria indicates that the maximum stress intensifier that normally needs to be used in evaluating intensified stress is a factor of approximately 5, except for notch-like defects. When this information is considered with the documentation of the acceptable dynamic strain/stress given by Wachel (1982), it is possible to determine an acceptable vibration limit from the stress per mil value for the piping span. An acceptable level of dynamic strain for most piping systems is 50 x 10^{-6} in/in, zero to peak. Based on this dynamic strain and a modulus of elasticity of 30 x 10^6 psi, it can be seen that if the combined stress intensity factors C_2K_2 are 5.12, the allowable dynamic strain would be 50 x 10^{-6} in/in which is equal to 1500 psi, zero to peak stress amplitude or 3000 psi, peak to peak stress range. The normal measurement for vibration displacement (deflection) is in mils, peak to peak, therefore, the allowable vibration displacement in mils peak to peak can be determined directly from the stress per mil scale, if the value of 5.12 for C_2K_2 is used.

Therefore, once the stress per mil is determined, a normally conservative value of the acceptable vibration displacement can be determined at the same location on the stress per mil scale by reading on the opposite side of the scale where the allowable vibration amplitude is labeled. This scale assumes that the C_2K_2 value is 5.12, and is the maximum that has to be used. If it is known that the value of C_2K_2 is less than 5.12, then the allowable vibration level read from the nomogram (y_{all-n}) can be ratioed by 5.12 over the actual C_2K_2 values. Since most piping spans will have fairly high safety factors, it normally is faster to determine a conservative allowable vibration amplitude and compare the measured value. Most piping spans will have maximum vibration levels lower than the conservative allowable vibration amplitude given on the scale, therefore, the acceptability of the span vibration and stress amplitudes are documented.

$$y_{all} = y_{all-n}\left(\frac{5.12}{C_2K_2}\right) \qquad (21)$$

Deviations in the Span Natural Frequency

Since the basis of the characteristic spans for Vibration Monitoring Group 2 assumes that the piping span is vibrating at its first lateral natural frequency, the nomogram also makes this assumption. Therefore, the results of the stress calculations can not be verified unless the measured natural frequency matches the calculated natural frequency. The derivation of the stress deflection factors discussed above indicates that the deflection stress factors can be adjusted if the measured natural frequency does not match the calculated natural frequency. It is known that differences in the boundary conditions from the ideal end conditions can affect the natural frequency and dynamic stress. If the basic mode shape of the piping span vibration matches the expected mode shape, but, the natural frequency deviates from the calculated natural frequency by ± 25 percent, the stress deflection factor can be adjusted by the ratio of the measured natural frequency to the calculated natural frequency.

$$(K_d)_{meas} = (K_d)_{cal}\left(\frac{f_{meas}}{f_{cal}}\right) \qquad (22)$$

Using the new value of the deflection factor $\left[(K_d)_{meas}\right]$, connect this value to the point on the reference line and obtain the stress per mil value and the conservative allowable vibration amplitude.

Piping Spans With Weights

The nomogram can be used to calculate the stress per mil and the allowable vibration amplitude for piping spans with concentrated weights by using the stress deflection factors and the point on the reference line. The correction factors that can be used to correct the natural frequency calculations for the concentrated weights are given in Figure 4. When using the displacement method for calculating stresses, the correction factor for the use of concentrated weights changes linearly from 1 to 2 as the ratio of the concentrated weight to the span weight increases from 0 to 20. For most practical cases, the weight ratio will result in a stress weight correction factor of less than 1.3.

Note that the span natural frequency for spans with concentrated weights can be calculated using the weight correction factors given in Figure 4.

Allowable Velocity Amplitude

The allowable vibration velocity can be calculated by calculating the allowable vibration displacement and converting this value to velocity by the relationship between the displacement and velocity. The allowable velocity in in/sec (V_{all}) is equal to the allowable vibration displacement times the natural frequency divided by 318.3. Using this value, the vibration measurement could be made in velocity units.

$$V_{all} = y_{all}\left(\frac{f}{318.3}\right) \qquad (23)$$

CONCLUSIONS

Based on the theory used in ASME Operations and Maintenance Standards/Guides Part 3 for evaluating the piping span vibration under steady state operation, a nomogram has been developed which provides help in implementing the procedures. The nomogram allows the analyst to check to see if the measured natural frequency matches the calculated natural frequency of the characteristic span. This is needed to verify that the assumptions used to calculate the vibration-induced stresses are valid. Information is provided on the nomogram for additional characteristic spans to cover piping configurations which are not now included in OM-3. The stress per mil for practically any type of piping configuration and boundary conditions can be calculated. The maximum dynamic stress can be calculated using the measured maximum vibration displacement and the intensifiers C_2K_2. Based on a normally conservative value of C_2K_2 of 5.12, the maximum allowable vibration amplitude can be read directly from the scale directly across from the stress per mil value. This nomogram should allow the analyst to quickly evaluate complex piping systems to determine which of the spans have an adequate margin of safety.

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REFERENCES

"Preoperational and Initial Startup Vibration Testing of Nuclear Power Plant Piping Systems," 1991, ANSI/ASME Operations & Maintenance Standards/Guides Part-3, New York, NY.

Blevins, R. D., 1979, "Formulas for Natural Frequency and Mode Shape," Van Nostrand Reinhold Company, New York, NY.

Olson, David E., 1985, "Piping Vibration Experience In Power Plants," *Pressure Vessel and Piping Technology 1985, A Decade of Progress*, Book No. H00330, (ASME).

"Positive Displacement Pumps, Reciprocating, API Standard 674," 1987, American Petroleum Institute, Washington, D.C.

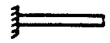
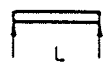
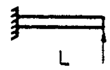
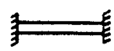
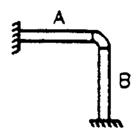
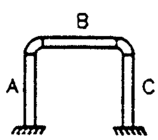
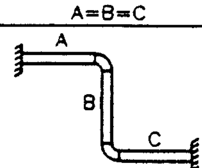
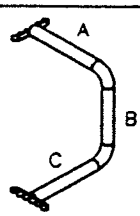
Wachel, J. C., 1992, "Field Investigation of Piping System for Vibration – Induced Stress and Failures," *Pressure Vessel and Piping Conference*, ASME Bound Volume No. H00219.

Wachel, J.C., Morton, S. J., and Atkins, K. E., 1990, "Piping Vibration Analysis," *Proceedings of 19th Turbomachinery Symposium*, Texas A&M University, College Station, TX.

Wachel, J. C., Szenasi, F. R., et. al., 1993, "Vibrations in Reciprocating Machinery and Piping," Engineering Dynamics Incorporated Report 41450.

GENERATION MECHANISM	DESCRIPTION OF EXCITATION FORCES	EXCITATION FREQUENCIES
1. MECHANICAL INDUCED		
A. Machinery Unbalanced Forces & Moments	High Level, Low Frequency	$f_1 = \frac{1N}{60}$ $f_2 = \frac{2N}{60}$
B. Structure - Bourne	Low Level	$f = \frac{N}{60}$
2. PULSATION INDUCED		
A. Reciprocating Compressors	High Pressure Pulsations, Low Frequency	$f = \frac{nN}{60}$ $n = 1,2,3,...$ (modes) N = Speed, rpm
B. Reciprocating Pumps	High Pressure Pulsations, Low Frequency	$f = \frac{nNP}{60}$ P = Number of Pump Plungers $f = \frac{nN}{60}$
C. Centrifugal Compressors & Pumps	Low Pressure Pulsations, High Frequency	$f = \frac{nBN}{60}$ B = Number of Blades $f = \frac{nvN}{60}$ v = Number of Volutes or Diffuser Vanes
3. GASEOUS FLOW EXCITED		
A. Flow Through Pressure Letdown Valves or Restrictions/Obstructions	High Acoustic Energy, Mid to High Broad Band Frequencies	$f = S\frac{V}{D}$ S = Strouhal Number 0.2-0.5 V = Flow Velocity ft/sec D = Restriction Diameter, ft.
B. Flow Past Stubs	Moderate Acoustic Energy Mid to High Frequencies	$f = S\frac{V}{D}$ S = 0.2 - 0.5 D = Stub Diameter, ft.
4. LIQUID (OR MIXED PHASE) FLOW EXCITED		
A. Flow Turbulence Due to Quasi Steady Flow (e.g. Fluid Solids Lines)	Random Vibrations, Low Frequency	$f = 0 - 30\ H_z$ (Typically)
B. Cavitation and Flashing	High Acoustic Energy, Mid to High Frequencies	Broad Band
5. PRESSURE SURGE/ HYDRAULIC HAMMER	Transient Shock Loading	Discrete Events

FIGURE 1
PIPING VIBRATION EXCITATION SOURCES

	Piping Configuration	Frequency Factor		Deflection Stress* Factor		Velocity Stress* Factor	
		1st	2nd	1st	2nd	1st	2nd
	Fixed-Free	3.52	22.4	366	2295	219	219
	Simply Supported	9.87	39.5	1028	4112	219	219
	Fixed-Supported	15.4	50.0	2128	6884	290	290
	Fixed-Fixed	22.4	61.7	2935	8534	275	290
 A+B = L A=B	L-Bend Out	16.5	67.6	1889	9690	241	301
	L-Bend In	59.4	75.5	7798	9575	276	266
 A+B+C = L A=B=C	U-Bend Out	18.7	35.6	2794	4628	314	273
	U-Bend In	23.7	95.8	3751	8722	332	191
 A+B+C = L A=B=C	Z-Bend Out	23.4	34.2	3522	4133	317	254
	Z-Bend In	22.4	96.8	3524	8933	331	194
 A+B+C = L A=B=C	3-D Bend	20.6	27.8	3987	4752	407	359
	Formula	$f = 75.8\lambda \frac{D}{L^2}$		$S = K_d Y \frac{D}{L^2} C_2 K_2$		$S = K_v V C_2 K_2$	

*Steel Piping ($E=30 \times 10^6$ psi, $\rho = 0.283$ lb/in^3)

FIGURE 2 FREQUENCY FACTORS AND STRESS FACTORS FOR UNIFORM STEEL PIPE CONFIGURATIONS

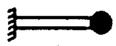
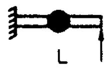
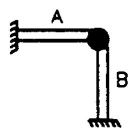
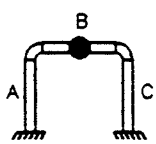
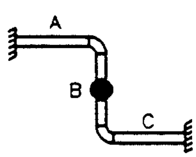
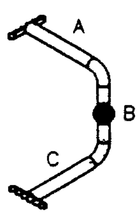
$f_p = \frac{f_o}{\sqrt{1+\alpha\frac{P}{W}}}$	Piping Configuration	Weight Location	Weight Correction Factor α
	Cantilever	L 3L/4	3.9 1.7
	Simply Supported	L/2 L/4	2.0 1.1
	Fixed-Supported	L/2 3L/4	2.3 1.6
	Fixed-End	L/2 L/4	2.7 0.9
 A+B = L A=B	L-Bend	A/2 A B/2	First Mode Out-of-Plane B/A 0.5 — B/A 1.0 1.24 — 0.63 2.69 — 3.25 0.39 — 0.63
 A+B+C = L A=B=C	U-Bend	A/2 A B/2	First Mode B/A = 1.0 Out — In 0.26 — 0.33 2.24 — 1.79 2.31 — 2.00
 A+B+C = L A=B=C	Z-Bend	A/2 A B/2	First Mode B/A = 1.0 Out — In 0.40 — 0.29 2.76 — 1.77 2.21 — 2.09
 A+B+C = L A=B=C	3-D Bend	A/2 A B/2	First Mode B/A = 1.0 1st — 2nd 0.31 — 0.58 2.35 — 1.58 2.06 — 2.77

FIGURE 3 WEIGHT CORRECTION FACTORS AND UNIFORM PIPING CONFIGURATIONS

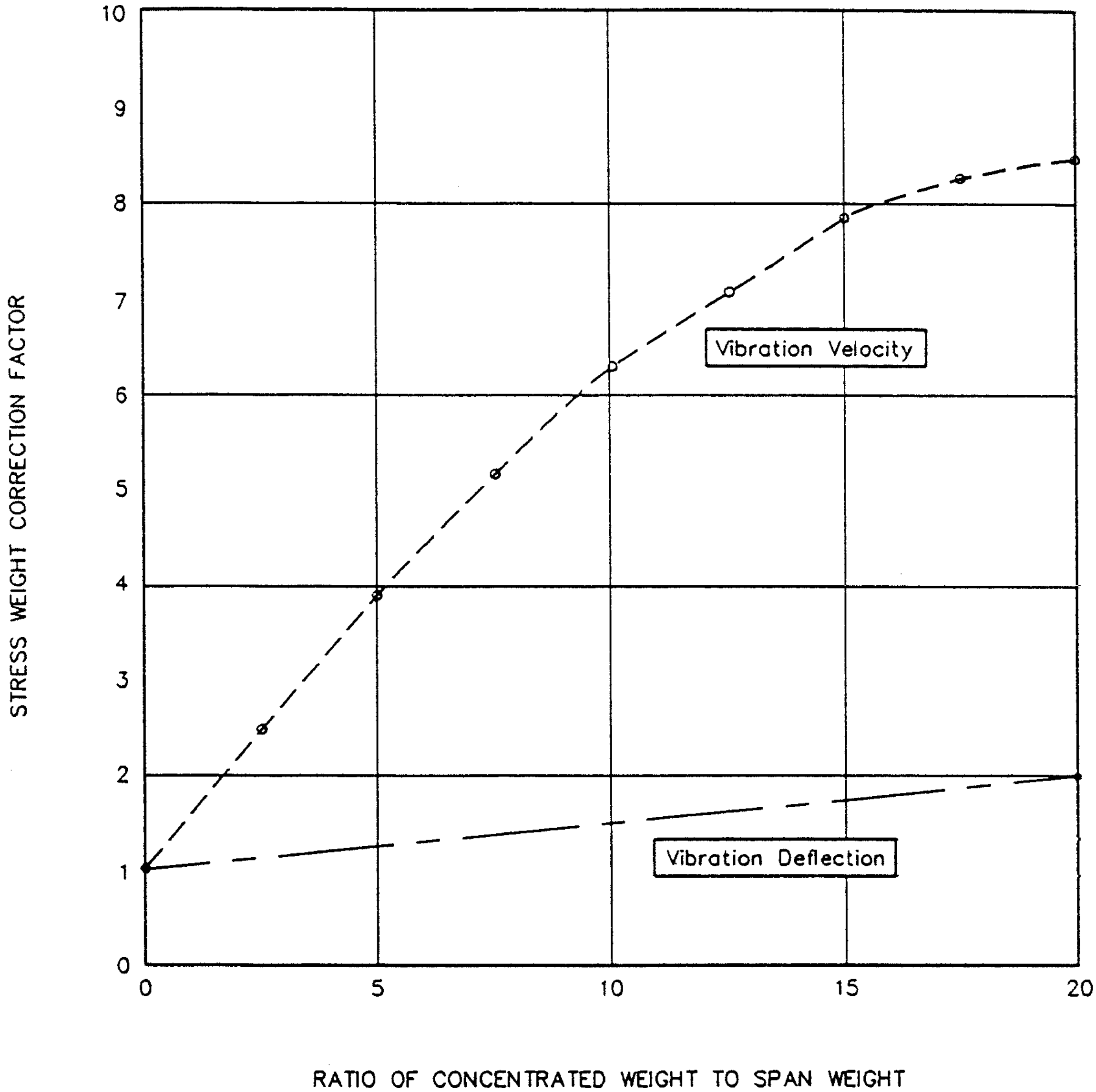


FIGURE 4 STRESS WEIGHT CORRECTION FACTORS

FIGURE 5

NATURAL FREQUENCY, VIBRATION-INDUCED STRESS & ALLOWABLE VIBRATION OF PIPING SPANS
For ASME OM S/C Part 3 Procedures Using Displacement Method

Frequency factors are based on first natural frequency of piping span

Allowable piping vibration based on OM-3 Stress Allowable of 7690 psi o-p with $C_2 K_2$ of 5.12