Technical Article

Least Squares Balance Program for the Hewlett-Packard 48GX Calculator

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Summary: The least-squares method is used for balancing multispeed, multiplane rotating machinery. A balance program for the HP-48GX is presented with an example problem.

The least squares method is commonly used for calculating balance corrections. In 1964, when Goodman [1] published his method, the computations had to be made on a main frame computer. The process often involved telephoning from a remote location to a computer center and waiting for results. Today, handheld calculators are capable of performing these calculations. Fielding and Mondy [2] wrote a balance program for a Texas Instruments 59 calculator in 1981. The program presented in this article is more general and performs both the plain and weighted least squares methods for n balance planes and m vibration readings. The vibration data may be taken at a number of locations and speeds. Multiple baselines can be used so that previous trial weights need not be removed.

Background

Shaft vibration readings are often measured with proximity probes and consist of amplitude and phase data. Any significant mechanical or electrical runout should be subtracted from the vibration readings before the balance calculations are carried out. The phase angles are determined from a once-per-revolution signal (key phasor).

The first step is to take baseline vibration data.

$$\{A\} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$$

Vibration data should not be taken until the machinery is fully heat soaked and the amplitude and phase at all locations remain constant.

The second step is to install a trial weight and take vibration readings. In general, the initial trial weight should not cause a force greater than 10% of the rotor weight at the operating speed. The angular location of the trial weight is referenced to the key phasor opposite shaft rotation. The trial weights at each plane are stored in the following complex vector.

$$\{U\} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

The vibration readings associated with each trial weight are stored in columns.

$$[B] = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix}$$

The response coefficients are calculated by subtracting the baseline from the trial data and dividing by the trial weight.

$$[\alpha] = \begin{bmatrix} (b_{11}-a_1)/u_1 & (b_{12}-a_1)/u_2 & \dots & (b_{1n}-a_1)/u_n \\ (b_{21}-a_2)/u_1 & (b_{22}-a_2)/u_2 & \dots & (b_{2n}-a_2)/u_n \\ \vdots & \vdots & & \vdots \\ (b_{m1}-a_m)/u_1 & (b_{m2}-a_m)/u_2 & \dots & (b_{mn}-a_m)/u_n \end{bmatrix}$$

The response coefficients are used to determine the correction weights needed to minimize the residual vibration.

$$\{\epsilon\} = \{A\} + [\alpha] \times \{W\}$$

The correction weight vector is

$$\{W\} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

The residual vibration vector is

$$\{\epsilon\} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_m \end{bmatrix}$$

Linear behavior is assumed for the least squares method. However, if the vibrations are high, the system could behave nonlinearly and a significant balance weight may be required to put the system back into a linear range.The installation of weights, data collection, and calculation are repeated until the vibration levels are acceptable or until no further improvement can be made. Improvement may not be possible if the number of balance planes is limited or a balance plane is not located in

an optimum location. Theoretically, the lowest vibration that can be expected from balancing is given by the RMS residual

$$R = \sqrt{\frac{S}{m}}$$

The sum of squared residuals is

$$S = \sum_{i=1}^{m} \epsilon_i^2$$

The weighted least squares method is similar to plain least squares except that iterations are used to weight the response coefficients by using the calculated correction weights and residual vibration from the previous iteration.

$$[\alpha'] = \begin{bmatrix} \alpha_{11}\left|\frac{\epsilon_1}{R}\right| & \alpha_{12}\left|\frac{\epsilon_1}{R}\right| & \cdots & \alpha_{1n}\left|\frac{\epsilon_1}{R}\right| \\ \alpha_{21}\left|\frac{\epsilon_2}{R}\right| & \alpha_{22}\left|\frac{\epsilon_2}{R}\right| & \cdots & \alpha_{2n}\left|\frac{\epsilon_2}{R}\right| \\ \vdots & \vdots & & \vdots \\ \alpha_{m1}\left|\frac{\epsilon_m}{R}\right| & \alpha_{m2}\left|\frac{\epsilon_m}{R}\right| & \cdots & \alpha_{mn}\left|\frac{\epsilon_m}{R}\right| \end{bmatrix}$$

The response coefficients used in the next iteration would be

$$[\alpha^I] = [\alpha']^T \times [\alpha]$$

Likewise, the baseline used in the next iteration would become

$$\{A^I\} = [\alpha']^T \times \{A\}$$

The balance calculations are carried out as with the plain least squares method, but $[\alpha^I]$ and $\{A^I\}$ are used in place of [α] and {A}. For the subsequent iteration,

$$[\alpha''] = \begin{bmatrix} \alpha'_{11}\left|\frac{\epsilon'_1}{R'}\right| & \alpha'_{12}\left|\frac{\epsilon'_1}{R'}\right| & \cdots & \alpha'_{1n}\left|\frac{\epsilon'_1}{R'}\right| \\ \alpha'_{21}\left|\frac{\epsilon'_2}{R'}\right| & \alpha'_{22}\left|\frac{\epsilon'_2}{R'}\right| & \cdots & \alpha'_{2n}\left|\frac{\epsilon'_2}{R'}\right| \\ \vdots & \vdots & & \vdots \\ \alpha'_{m1}\left|\frac{\epsilon'_m}{R'}\right| & \alpha'_{m2}\left|\frac{\epsilon'_m}{R'}\right| & \cdots & \alpha'_{mn}\left|\frac{\epsilon'_m}{R'}\right| \end{bmatrix}$$

$$[\alpha^I] = [\alpha'']^T \times [\alpha]$$

$$\{A^I\} = [\alpha'']^T \times \{A\}$$

Program

Goodman presented the least squares method using only one baseline. However, removing a weight from one balance plane before adding a weight to another balance plane is not always practical. Previous trial data can be used as baseline data for subsequent trial weights without removing weights. The calculated weights and angles are in addition to those installed when the minimized data were taken. This level of generality in the balance program requires that a baseline, trial weight, and trial data must be entered for each balance plane. In addition, the data to be minimized must be specified.

The HP 48GX calculator should be set up so that the angle measure is in degrees and the coordinate system is polar. The stack must be cleared before entering the data:

Baseline for Plane 1 (Complex Vector)
Weight for Plane 1 (Complex Number)
Trial Data for Plane 1 (Complex Vector)
Repeat Above Three Lines for n Planes
Data to Minimize (Complex Vector)

The BEGIN program takes information from the stack and sets up the variables to be used. The CALC program performs the balance calculations on the current variables. The ITER program updates and weighs the variables.The BAL program performs the weighted least squares method by using the other programs until the solution has converged. Note that only the BEGIN and CALC programs are required to perform plain least squares calculations. Program listings are given in Figure 1 through Figure 4.

```
≪ DUP 'A' STO 'AI' STO          Store the data to be minimized.
DEPTH DUP 3 / → n n3            Sort the remaining data on the stack.
≪ n n3 2 + FOR x
x ROLL -2 STEP
n n3 2 * 1 + FOR x
x ROLL -1 STEP
n3 COL→ 'W' STO                 Store the weights.
n3 COL→ 'BL' STO                Store the baseline data.
n3 COL→ 'TD' STO                Store the trial data.
TD BL - →COL →LIST              Calculate the response coefficients.
W →COL →LIST /
OBJ→ COL→ DUP DUP
'α' STO 'αI' STO                Store the response coefficients.
'αP' STO ≫ ≫
```

Figure 1. Program Listing for BEGIN.

```
≪ AI NEG αI LSQ                 Calculate and store the correction weights.
'WI' STO
A α NEG WI RSD                  Calculate and store the residual vibration.
DUP 'ϵ' STO
ABS SQ DUP 'S' STO              Calculate and store the sum of squared residuals.
ϵ SIZE OBJ→ DROP                Calculate and store the RMS residual.
/ √ 'R' STO ≫
```

Figure 2. Program Listing for CALC.

Example

A centrifugal compressor has two balance planes of the same size (forward and aft). A balance weight of 10.2 grams was already located on the aft balance plane at 66°. Baseline vibration readings were taken at the four shaft proximity probes while the compressor was at full speed. Next, the balance weight on the aft balance plane was changed to a total of 20.5 grams at 50° and trial vibration readings were taken. Finally, a balance weight of 3.7 grams was installed on the forward balance plane at 135° and the trial vibration readings were taken again. The vibration readings are listed in the Table.

Vibration Readings with Various Balance Weights.

Balance Weights (grams)		Vibration Readings (mils p–p)			
Fwd	Aft	Fwd X	Fwd Y	Aft X	Aft Y
	10.2 @ 66°	0.68 @ 32°	0.56 @ 86°	1.94 @ 231°	2.07 @ 335°
	20.5 @ 50°	1.31 @ 1°	1.25 @ 75°	0.93 @ 251°	1.00 @ 342°
3.7 @ 135°	20.5 @ 50°	0.54 @ 9°	0.52 @ 75°	0.81 @ 196°	0.90 @ 296°

To determine the calculated correction weights using the plain and weighted least squares methods, clear the stack and enter the following:

```
[ (0.68∠32) (0.56∠86) (1.94∠231) (2.07∠335) ]
(11.1∠35)                    This weight is the difference between (20.5∠50) and (10.2∠66).
[ (1.31∠1) (1.25∠75) (0.93∠251) (1.0∠342) ]
[ (1.31∠1) (1.25∠75) (0.93∠251) (1.0∠342) ]
(3.7∠135)
[ (0.54∠9) (0.52∠75) (0.81∠196) (0.9∠296) ]
[ (0.68∠32) (0.56∠86) (1.94∠231) (2.07∠335) ]
```

For plain least squares, run BEGIN then CALC.The iterative weight variable, WI, is equal to [(15.3∠3) (6.6∠113)]. Therefore, the calculated correction weights are (15.3∠3) + (10.2∠66) = 21.9 grams at 28° for the aft plane and 6.6 grams at 113° for the forward plane. The calculated RMS residual, R, is 0.07. The calculated residual vibrations are stored in ϵ and are equal to [(0.08∠138) (0.09∠49) (0.05∠231) (0.05∠166)].

For weighted least squares, run BAL after the data are entered onto the stack. After 13 iterations, the calculated correction weights are (15.2∠4) + (10.2∠66) = 21.9 grams at 28° for the aft plane and 6.7 grams at 114° for the forward plane. The calculated RMS residual is 0.08 and the calculated residual vibrations are [0.08∠141) (0.08∠45) (0.08∠184) (0.08∠212)]. This example shows that the results were similar using either the plain or weighted least squares methods. For most cases the plain least squares method is sufficient to predict the required correction weights.

References

1. Goodman, Thomas P., "A Least-Squares Method for Computing Balance Corrections," J. Engrg. Indus., Trans. ASME, pp 273-279 (Aug 1964).
2. Fielding, L and Mondy, R.E., "A Better Way to Balance Turbomachinery," Hydrocarbon Proc., pp 97-104 (Jan 1981).


```
≪ ϵ OBJ→ OBJ→ DROP          Calculate the prime response coefficients.
→LIST ABS R /
αP →ROW →LIST *
OBJ→ ROW→ DUP 'αP' STO      Store the prime response coefficients.
TRN DUP α * 'αI' STO        Calculate and store the iterative response coefficients.
A * 'AI' STO ≫              Calculate and store the iterative data to be minimized.
```

Figure 3. Program Listing for ITER.

```
≪ CLLCD
"Balance Program" 4 DISP
"Initialize" 6 DISP
BEGIN CALC 0 → x
≪ DO "Iteration #"
'x' INCR →STR
+ 6 DISP
ϵ ITER CALC ϵ - ABS          Calculate the difference in residual vibration.
UNTIL .001 < END             Continue iterating until the difference is less than 0.001.
"DONE" 6 DISP ≫ ≫
```

Figure 4. Program Listing for BAL.

List of Variables.

A	Beginning Minimized Data
AI	Iterative Minimized Data
W	Beginning Weights
WI	Iterative Weights
α	Beginning Response Coefficients
αI	Iterative Response Coefficients
αP	Prime Response Coefficients
BL	Baseline Data
TD	Trial Data
ϵ	Residual Vibration
S	Sum of Squared Residuals
R	RMS Residual