BALANCE THIS!

CASE HISTORIES FROM DIFFICULT BALANCE JOBS

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ABSTRACT

There are many good references for classical balance problems; however, specific ways of handling non-ideal conditions are rarely presented. Therefore, this tutorial focuses on six case histories, which illustrate actual problems that were encountered while trying to field balance different types of rotating machinery. Coupling lockup, thermal bows, eccentricity, looseness, and structural resonances can prevent successful balancing if not properly identified and corrected. This tutorial shows how these real world pitfalls can complicate balance procedures and provides some ways of dealing with these issues such as: indexing coupling parts, using inference fits, adding structural bracing, and acquiring vibration measurements at additional locations. This tutorial also gives a brief description of the common types of unbalance.

INTRODUCTION

Generally, when machinery experiences high vibration at 1× running speed, the first course of action that a person may take is to improve the balance of the rotor. This tutorial provides a brief overview of different unbalance conditions, theories, and criteria used when balancing rotating equipment. There are many good books, papers, and articles that explain balancing procedures in great detail as listed in the Bibliography. These works may also cover the limitations of various methods and provide warnings which state that vibration problems due to misalignment or instability can not be "balanced out." A literature search uncovered papers by Rieger (1984) and Baxter and Whaley (1995) containing case histories where balancing proved difficult. It is the intent of this tutorial to show how instances where balancing was not initially successful can be addressed. The six case histories demonstrate many of the pitfalls that can occur when trying to balance rotating equipment.

The purpose of balancing a rotor is to help ensure that the machinery is safe and reliable. This is achieved when the rotor mass and rotational centerlines are as close to equal as possible. Excessive unbalance can cause vibration and stress in the shaft or attached pieces. For example, a generator with high shaft vibration can cause the overhung exciter to loosen and fail. Dynamic forces can also be transmitted to supporting structures, such as the bearing housing or equipment case. An unbalanced centrifuge on a common deck can cause vibration in adjacent pieces of equipment. If operators have to walk on vibrating grating, this can cause annoyance and fatigue. Excessive vibration can cause wear in bearings, seals, gears, etc. and reduce the life expectancy of these parts.

As a rotor spins, centrifugal forces act upon it. The surface around the periphery is stressed as particles are pulled outward from the axis of rotation. If all of these radial forces are equal, the rotor is said to be balanced and should not vibrate. However, if the rotor contains a heavy spot to one side, these radial forces will not cancel and the unbalanced force will tend to pull the rotor from the center of rotation. Figure 1 shows that the centrifugal force produced by unbalance depends on the mass and radius as well as the rotational speed of the rotor.

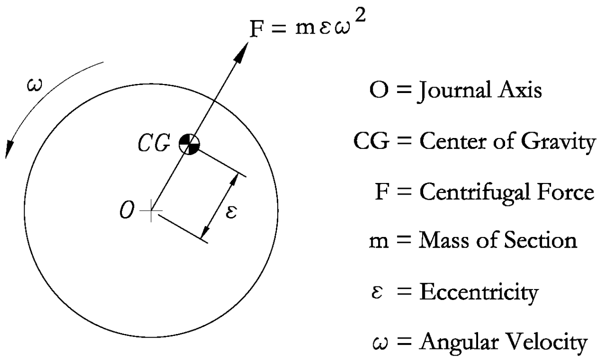

Figure 1: Rotor Section with CG Displaced Due to Unbalance

The physical conditions that can result in unbalance include:

- Variation of material density due to voids, porosity, or finish.
- Tolerances in fabrication, casting, machining, or assembly.
- Unsymmetrical parts such as a motor winding or built-up rotor.
- Shifting of parts due to shaft distortion, shrink fit, aerodynamic forces, or thermal effects.

TYPES OF UNBALANCE

As stated in Theory of Balancing (1973), ISO Recommendation No. 1925 lists four primary types of unbalance conditions: static, couple, quasi-static and dynamic. An unbalance or balance weight is expressed in terms of a mass multiplied by the radius. Typical English units would be ounce-inches, or in metric units, gram-mm. There are also occasions where the measurement systems could be mixed such as gram-inches depending on the type of scale or weights that are available. Figure 2 gives an example of two equivalent unbalances.

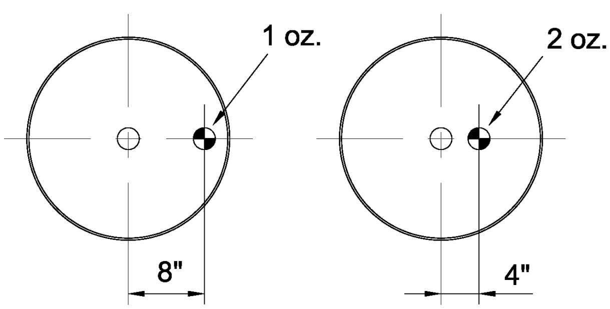

Figure 2: Example of Equivalent Unbalance Conditions

Static Unbalance

The word static refers to something that is stationary. Large static unbalance can be detected with a gravity type balancing method where a rotor is placed on “knife-edges.” For this test, the rotor is not spinning, but will tend to roll so that the heavy spot is downward. Static unbalance may occur in thin, disc-shaped parts such as impellers, flywheels, and fans.

Figure 3 shows how the principal axis of inertia differs from the axis of rotation, but is still parallel. Thus, a pure static unbalance can be corrected with a single balance weight. An eccentric rotor can be also thought of as creating a static unbalance condition. In addition, if a rotor contained two or more equal unbalances that could be combined into an equivalent unbalance at the center-of-gravity (CG), this too would be considered static unbalance since the condition could be corrected with a single weight.

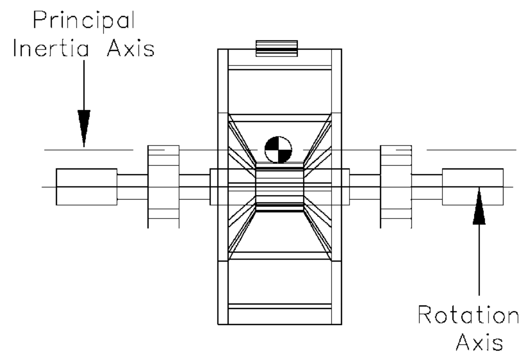

Figure 3: Example of Static Unbalance

Recall that the centrifugal force due to unbalance increases with operating speed squared. Therefore, a static unbalance could be more accurately determined while the rotor is rotating as opposed to using gravitational methods.

Couple Unbalance

A couple unbalance occurs when two equal unbalances are 180° out-of-phase and on opposite ends of a rotor. Figure 4 shows that for this condition, the principal axis of inertia will intersect the rotational axis at the rotor CG. If placed on knife-edges, a rotor with pure coupling unbalance will not roll to a heavy spot since no static unbalance is present. Therefore, a dynamic method of balancing must be used while the rotor is spinning. The moment forces will produce vibration with different phase angles at each bearing thus giving an indication of couple unbalance.

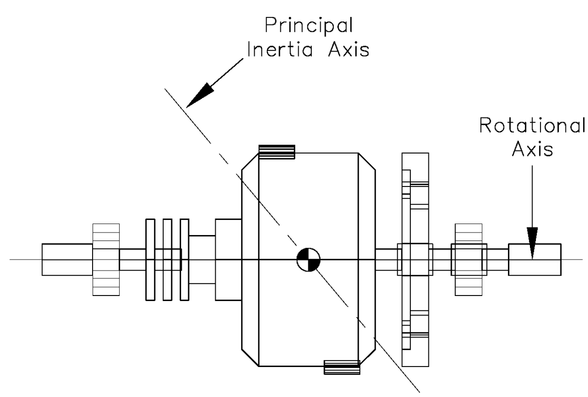

Figure 4: Example of Couple Unbalance

Note that this type of unbalance is not correctable with just a single weight. To create a counter acting couple, at least two weights in two separate planes are required. Again, the equivalent balance weights and proper couple do not have to match the exact locations of the unbalance and could be produced by many combinations of mass, radius, and axial position along the shaft.

Quasi-Static Unbalance

There are cases where a single weight or combination of weights may appear to create a static unbalance; however, the principal axis of inertia intersects the rotational axis of the rotor at a point that does not coincide with the CG. Quasi-static unbalance can occur due to the axial location of the unbalance mass(es) on the rotor.

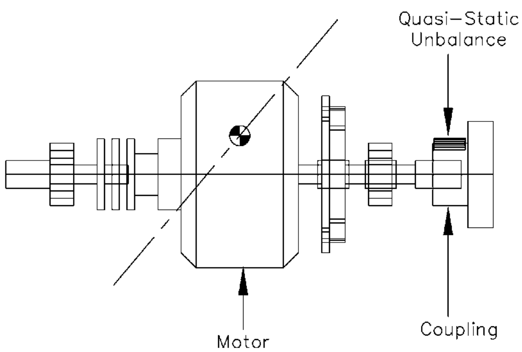

Figure 5: Quasi-Static Unbalance

For example, an unbalanced coupling that is overhung from the drive-end bearing can be considered a quasi-static unbalance condition (Figure 5). Combinations of static unbalance and couple unbalance where the angular positions of the weights coincide can also create quasi-static unbalance in a rotor, which is really a subset of dynamic unbalance.

Dynamic Unbalance

The unbalance types reviewed so far have been simplified examples. In general, unbalance of a rotor may be located at numerous locations. Dynamic unbalance occurs frequently in machinery such as multi-staged compressors or steam turbines and must be corrected using at least two balance planes. Figure 6 shows that for dynamic unbalance, the principal axis of inertia is not coincident with the rotational axis.

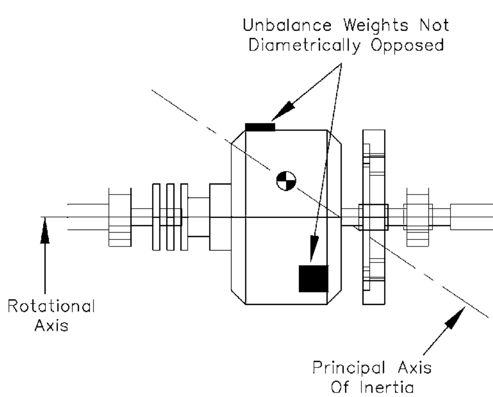

Figure 6: Dynamic Unbalance

EFFECTS OF LATERAL CRITICAL SPEEDS

The relationship of the location of the unbalance force with respect to the shaft center and rotational axis is greatly affected by the proximity of the shaft speed to the lateral natural frequency. Recall that the amount of unbalance is defined as the mass of the disc times the eccentricity of the center-of-gravity (CG). Well below the critical speed, the high side of the whirling shaft coincides with the angular location of the CG (Figure 7a).

As the running speed of the rotor approaches the critical speed, the center of rotation begins to shift toward the CG (Figure 7b). At resonance, the phase angle between the exciting force (direction of the unbalance) and the actual vibration (high side) will be 90°. In this particular case, the vibration amplitude, or high side, lags the unbalance by 90°.

As the shaft speed passes through the critical speed, the phase angle increases to 180° as the disc tries to rotate about its CG, which is no longer concentric with the geometric center (Figure 7c). Szenasi, Smith, et al. (1996) have written several chapters that provide further explanation on basic vibration theory, lateral critical speeds, rotor response, and balancing.

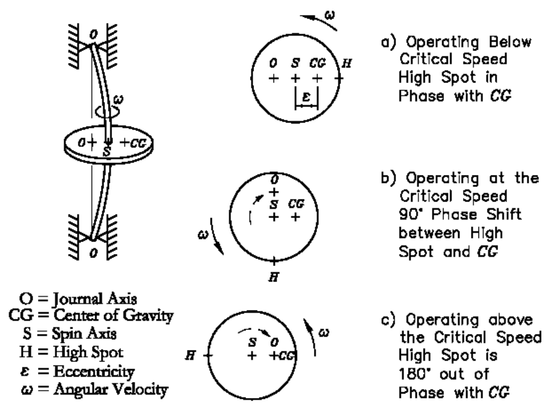

Figure 7: Phase Relationship of Unbalance and Vibration When Operating Near Critical Speed

REVIEW OF BALANCING METHODS

Due to space limitations, it is not practical to cover all of the available balancing methods in great detail. Therefore, this short review highlights the most common balancing methods used in industry. The reader is also encouraged to refer to the papers and books listed in the References and Bibliography sections of this tutorial.

Vector Method

For many years, balancing has been performed using graphical techniques as shown in Figure 8. Jackson (1991) showed how to perform single-plane balancing by plotting vibration readings (amplitude and phase) as vectors on polar graph paper. Balance corrections were computed by determining the difference between baseline and trial weight vectors and then scaling the resultant to obtain the proper location and weight.

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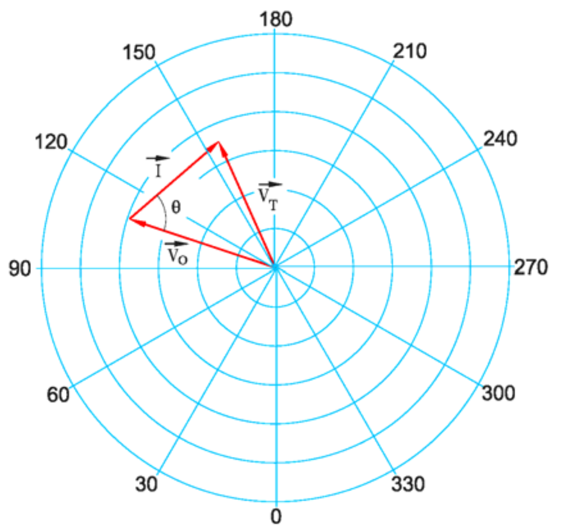

Figure 8: Single-Plane Vector Solution

For example, vector V_O represents the original vibration reading and V_T is the vibration after the trial weight was applied. After these two vectors are plotted, vector I should be constructed by connecting V_O to V_T. By taking the ratio of the lengths of vectors V_O and I, the correction weight can be computed as:

$$\text{Correction Weight} = \text{Trial Weight} \times V_O / I.$$

Using a protractor the included angle, θ, between vectors V_O and I can be measured. The computed correction weight should be located at this angle from the initial trial weight position and the trial weight should be removed. The direction of the shift will depend upon the reference system used.

Currently, programmable calculators are capable of performing more complicated balancing methods such as influence coefficients. For example, Fielding and Mondy (1981) and Feese (1998a) developed balance programs for calculators. However, it is still a good idea to draw out the vectors by hand to help check the calculations.

Four-Run Method

When phase data is unavailable, single-plane balancing can still be performed using the four-run method. This method is often used on fans and slow speed equipment. There are other instances where the four-run technique may also be valuable such as when nonlinearities are present in the system.

The basic steps of the four-run method are as follows:

1) The original or baseline vibration (Run 0) is drawn as a circle instead of a vector since phase angles are not recorded. The vibration amplitude determines the radius of the circle.

2) For the next three runs, a trial weight is placed on the rotor at 120° increments. The vibration amplitude for each trial run is measured and then represented with an additional circle centered at 120° increments around the original circle in the diagram.

3) At the end of the procedure, the three circles from the trial weight runs should have a common intersection point. If the circles do not exactly intersect, a "guesstimate" should be made as to the location of the intersection point.

4) The distance from the center of the original circle to the intersection point helps determine the correct balance weight needed for the final correction. The angular location for placement of the weight is indicated by the angle of the intersection vector.

An example diagram is shown in Figure 9. The radius of Run 0 represents the original vibration reading of 0.088 inches per second (ips). A trial weight of 7 grams (0.25 oz) was then placed on the rotor at 120° increments. The resulting vibration levels from Run 1 through Run 3 were also drawn as circles.

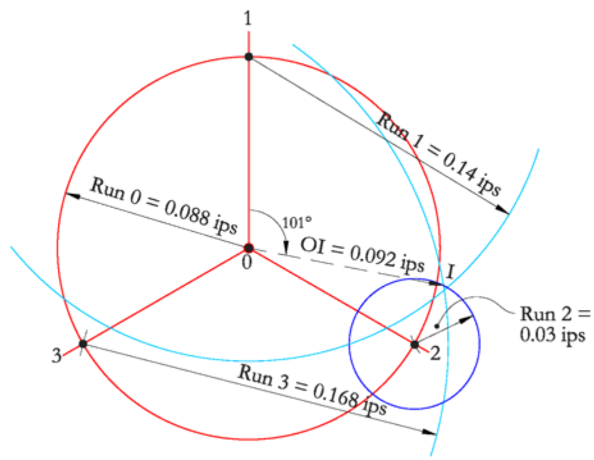

Figure 9: Four-Run Solution

A vector was drawn from point 0 to I, which is the approximate intersection point of the four circles. The measured length of vector 0I represented 0.092 ips. The correction weight was computed by dividing the radius from Run 0 by the length of vector 0I:

$$\text{Correction Weight} = \text{Trial Weight} \times \text{Run 0} / 0I$$
or
$$6.7 \text{ grams} = 7 \text{ grams} \times 0.088 \text{ ips} / 0.092 \text{ ips}$$

The included angle from the first trial weight location to vector 0I was 101°. The computed correction weight should then be located at this offset angle from the initial trial weight position.

Modal Balancing

As discussed previously, rotors have lateral critical speeds that can affect vibration amplitudes and phase angles. At resonance, the rotor deflection will be similar to the mode shape of the lateral natural frequency, which depends on the physical characteristics of the rotor and the bearing properties. Using simplified beam theory where the shaft is flexible compared to the support stiffness, the undamped mode shapes at the first three lateral critical speeds resemble those shown in Figure 10.

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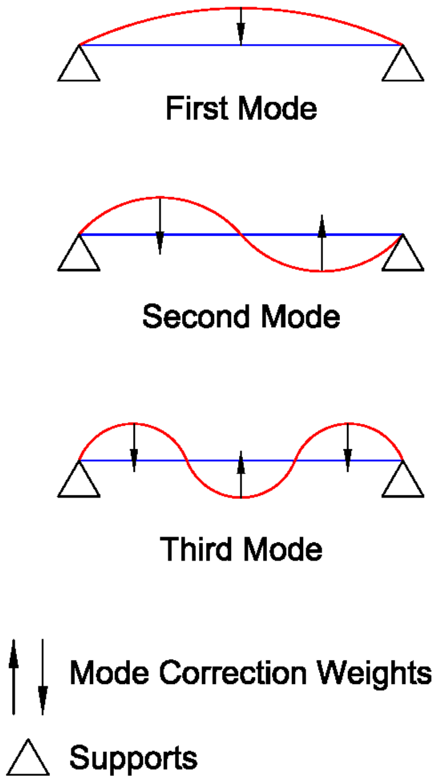

Figure 10: Basic Flexible Modes and Correction Weights.

Nodes are locations where minimal shaft movement is expected for a particular mode. For the first mode shown, there are two nodes, one at each support location. The midspan of the rotor has the highest relative amplitude, which is at the anti-node. The response of a rotor running near a critical speed will usually correspond to the mode shape provided the unbalance is located at the proper location. Therefore, unbalance near the center of the rotor would be expected to cause the most vibration due to the bowed shape of the first mode. However, unbalance near the supports would create very little vibration. The actual vibration amplitudes at resonance will depend on the unbalance and damping in the system.

Darlow (1989) provides an excellent source of information on the concept of modal balancing, which is to address each mode of concern independently. The ideal location for balancing the first mode would be to have a plane at the midspan of the rotor. Theoretically, applying the proper balance weight at that location would yield the best results for the first mode and should not affect the second mode. This would be important to consider when optimizing a machine running in a speed range between the first and second modes. Unfortunately in the real world, balance planes are often not where you would prefer for them to be.

Influence Coefficient Balancing

Modal balancing requires some knowledge of the lateral critical speeds and mode shapes. This would be available from a rotordynamic study. On the other hand, influence coefficients can be determined from trial weights and vibration measurements. Multiple coefficients can be obtained by applying trial weights independently at different planes and operating speeds. Since redundant information can be gathered from multiple accelerometers or probes, the least squares method may be used to statistically average the data for computation of the influence coefficients.

Goodman (1964) published a well known paper on the least squares method for computing balance corrections, which has since been implemented on computers and calculators. Variations include plain and weighted least squares methods for 'n' balance planes and 'm' vibration readings. The vibration data may be taken at a number of locations and speeds. Multiple baselines can be used so that previous trial weights do not have to be removed.

Shaft vibration readings are often measured using proximity probes and consist of amplitude and phase data. If significant mechanical or electrical runout exists, it should be subtracted from the vibration readings before performing the balance calculations. The phase angles are determined from an once-per-revolution signal (key phase).

The first step is to obtain baseline vibration data. Vibration data should not be taken until the machinery is fully heat soaked and the amplitude and phase at all locations stabilize.

Next, a trial weight is installed and vibration readings are taken. A good rule of thumb is that the initial trial weight should not cause a force greater than 10 percent of the rotor weight at the operating speed. The angular location of the trial weight is referenced to the key phase and is measured opposite shaft rotation.

The response coefficients are calculated by subtracting the baseline from the trial data and dividing by the trial weight. The response coefficients are used to determine the correction weights needed to minimize the residual vibration.

Linear behavior is assumed for the least squares method. However, if the vibrations are high, the system could behave nonlinearly and a significant balance weight may be required to put the system back into a linear range.

The procedures for installing weights, collecting data, and calculating corrections are repeated until the vibration levels are acceptable, or until no further improvement can be made. Improvement may not be possible if the number of balance planes is limited, or a balance plane is not located in an optimum location. Theoretically, the lowest vibration that can be expected from balancing is given by the root mean square (RMS) of the calculated residual vibration levels.

Goodman presented the least squares method using only one baseline. However, removing a weight from one balance plane before adding a weight to another balance plane is not always practical. Previous trial data can be used as baseline data for subsequent trial weights without removing weights. The calculated weights and angles are in addition to what was installed when the minimized data was taken. To accomplish this level of generality in the balance program, a baseline, trial weight, and trial data must be entered for each balance plane and the vibration data to be minimized must be specified.

Example Two-Plane Balance Problem

A centrifugal compressor has two balance planes of the same diameter (forward and aft). Baseline vibration readings were taken at the four shaft proximity probes (X and Y directions at the forward and aft ends) while the compressor was operating at 17,500 RPM. The shaft vibration on the aft end was higher than the desired level of 0.9 mil p-p (0.0009 inch peak-to-peak), so field balancing was performed.

A balance weight of 11.1 grams at 35° was added to the aft balance plane and trial vibration readings were taken. Then, a balance weight of 3.7 grams was installed on the forward balance plane at 135° and another set of vibration readings were taken. The vibration readings are listed in Table 1.

Table 1: Vibration Readings with Various Balance Weights

Balance Weights (g)		**Vibration Readings (mils p-p)**			
Fwd	Aft	Fwd X	Fwd Y	Aft X	Aft Y
Baseline Readings		0.68 @ 32°	0.56 @ 86°	1.94 @ 231°	2.07 @ 335°
---	11.1@35°	1.31 @ 1°	1.25 @ 75°	0.93 @ 251°	1.00 @ 342°
3.7 @ 135°	11.1@35°	0.54 @ 9°	0.52 @ 75°	0.81 @ 196°	0.90 @ 296°

Using the least squares method, the calculated correction weights were 15.3 grams @ 3° for the aft plane and 6.6 grams at 113° for the forward plane. The calculated RMS residual was 0.07 mils p-p. The calculated residual vibrations were:

Fwd X	0.08 mil p-p @ 138°
Fwd Y	0.09 mil p-p @ 49°
Aft X	0.05 mil p-p @ 231°
Aft Y	0.05 mil p-p @ 166°

Since the target vibration level of 0.9 mil p-p was achieved on the second trial run and it was difficult to shutdown the compressor and change weights due to the presence of hydrogen sulfide in the gas (sour gas), it was decided to suspend field balancing and accept the unit.

BALANCE SPECIFICATIONS

The test equipment used today is able to accurately measure vibration, even down to very low levels that would not be considered harmful to the machinery. Therefore, a reasonable balance limit must be set for each type of equipment so that long term reliability is achieved without undo expense of excessive balancing procedures. Several organizations publish recommended balance quality.

ASA STD-1975 (ISO Standard 1940)

In general, the larger the rotor mass or the slower the rotor speed, the greater the unbalance that can be tolerated. The balance quality grades (G) are based on experience with similar rigid rotors in various groups (Figure 11). Example classifications of these groups are shown in Table 2. The ISO specification applies to the entire rotor (total of all balance planes).

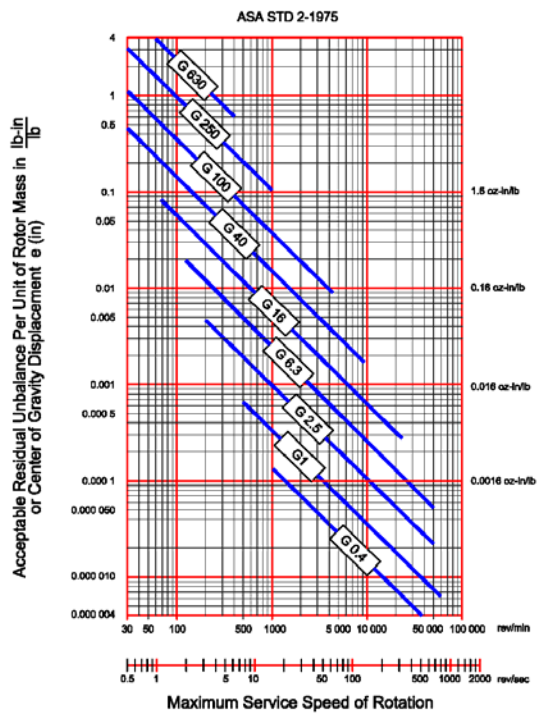

Figure 11: Balance Quality Grades

Table 2: Rotor Classifications

Rotor Classification (Balance Quality)	**Rotor Description (Examples of General Types)**
G 40	Passenger Car Wheels and Rims
G 16	Automotive Drive Shafts Parts of crushing and agricultural machinery
G 6.3	Drive shafts with special requirements Rotors of Processing machinery Centrifuge bowls; Fans Flywheels, Centrifugal pumps General machinery and machine tool parts Standard electric motor armatures
G 2.5	Gas and steam turbines, Blowers, Turbine rotor, Turbo generators, Machine tool drives, Medium and bigger electric motor armature with special requirements, Armatures of fractional hp motors, Pumps with turbine drive
G 1 Precision Balancing	Jet engine and super charger rotors Tape recorder and phonograph drives Grinding machine drives Armatures of fractional hp motors with special requirements

MIL-STD-167 (1974)

MIL-STD-167 is a military specification that deals with the mechanical vibration of shipboard equipment. Three formulas are listed in Table 3 for permissible unbalance depending on the operating speed.

The ISO specification discussed previously is based on experience factors, while the MIL standard is based on a percentage of the rotor weight. Note that the MIL specification applies to a single balance plane.

Table 3: MIL-STD-167

Speed Range, rpm	Max. Unbalance, oz-in
0 to 150	0.177W
150 to 1000	$4000\,\frac{W^2}{N}$
above 1000	$4\,\frac{W}{N}$

Where: W = total rotor weight (lbs)
N = maximum continuous speed (rpm)

API Standards

The American Petroleum Institute (API) also has standards for various types of turbomachinery in which balancing is addressed. For example, API 617 calls for major parts of the rotating element of a compressor, such as the shaft, balancing drum, and impellers, to be dynamically balanced. Keyways shall be filled with a fully crowned half-key when dynamically balanced. After the addition of no more than two major components, the rotating elements shall be multi-plane dynamically balanced during assembly.

The maximum allowable residual unbalance per plane (journal) shall be calculated as

$$U = \frac{4W}{N} \qquad (1)$$

where: U = residual unbalance, ounce-inches
W = journal static weight load, pounds
N = maximum continuous speed, RPM

Note that this appears to be the same equation as given in MIL-STD-167 for equipment operating above 1,000 RPM; however, W is the static weight load at the journal for API as compared to the total weight of the rotor specified in the MIL standard.

It is the authors' understanding that for a single midspan unbalance, the full rotor weight should be used when determining the API residual unbalance. For dynamic unbalance at two planes for a machine with two bearings, half of the rotor weight would be used at each location and applied 180 degrees out-of-phase. For unbalance of a coupling, the overhung weight of the shaft extension beyond the bearing and the coupling could be used for W in Equation 1 when computing the API residual unbalance.

API states that during the shop test of the machine, assembled with the balanced rotor, operating at its maximum continuous speed or at any other speed within the specified operating speed range, the peak-to-peak amplitude of unfiltered vibration in any plane, measured on the shaft adjacent and relative to each radial bearing, shall not exceed the following value or 2 mils, whichever is less

$$A = \sqrt{\frac{12000}{N}} \qquad (2)$$

where: A = unfiltered vibration level, mils p-p
N = maximum continuous speed, RPM

Ten Percent Force Method

When determining the initial size of the trial weight, the 10 percent force method may be a good rule of thumb to use if no other information is available. A robust bearing design should be able to withstand an additional 10 percent load due to dynamic force without affecting the bearing life. Wowk (1995) provides a simple form of the centrifugal force formula as

$$U = \frac{56347W}{N^2} \qquad (3)$$

where: U = residual unbalance, oz-in
W = journal weight, lbs

AVOIDING PITFALLS

In the field of vibration, many people wonder what percent is science or art. Experience can definitely play an important role when balancing a difficult piece of equipment. Paying close attention to detail is necessary for success.

General Problems That Can Affect Balancing

Some examples of pitfalls that can occur while balancing include:

- Measurement problems such as: bad location, probe runout (mechanical or electrical), erroneous signals due to damaged probes or cable connection.
- Looseness of hubs, bearings, or case footings (soft foot).
- Bent shaft, thermal distortion of rotor, or eccentric hub.
- Lateral critical speeds, wheel modes, and structural natural frequencies.
- Gear couplings that lockup under full torque.
- Excessive vibration or rubs that force amplitudes into a nonlinear range.
- Non-uniform buildup of debris or erosion of a rotating part.
- Reassembling a flange or coupling with bolts in different locations. Parts should be match marked or weighed.
- Misalignment due to imposed piping thermal loads (nozzle loads and/or case distortion).
- Balancing at low speed when vibration occurs at high speeds (mode shape).
- Balancing for high speed can sometimes cause problems when passing through the first lateral critical speed during startup and coastdown (mode shape).

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Initial Tasks That Should Be Performed Before Balancing

Before applying balance weights to machinery in the field, there are several tasks that should be performed to help identify some of the aforementioned problems. These include:

- Review lateral critical speed and unbalance response analyses, if available. Feese (1998b) showed how high shaft vibration of a synchronous generator was corrected with a rotordynamic analysis.
- Review previous balance reports.
- Obtain influence coefficients from manufacturer.
- When balancing using proximity probes, determine runout from slow-roll data so that it can be subtracted from readings during operation.
- Check for amplitude and phase repeatability between runs.
- Wait for the vibration vectors (amplitude and phase) to stabilize as rotor heat soaks. Some centrifugal compressors and large induced draft (ID) fans can take up to six hours and large steam turbines could take even longer. For example, MacPherson, Taylor, and Feese (2003) gave a case history for balancing an 11,000 RPM steam turbine.
- Check for critical speeds with waterfall or bode plots during startup or coastdown.
- Smith and Simmons (1980) recommend checking for wheel modes on large fans with impact tests while the unit is down.
- Check for structural natural frequencies with speed sweep, or shaker test. Take vibration survey or operating deflection shape data to determine motion of structure.
- Take baseline amplitude and phase readings at several locations with accelerometers and/or proximity probes. Extra readings can always be thrown out later, but may be valuable if some locations are not as sensitive as originally thought.
- Calculate a reasonable sized trial weight based on API, ISO balance quality grade, or 10 percent force method.
- Check the DC position of the shaft (shaft centerline plot) using proximity probes.
- Maintain the same operating condition (speed, load, etc.) for a balance run.

Potential Problems When Using the Least-Squares Method

The least-squares balance method is commonly used to compute influence coefficients from the baseline and trial weight data. The influence coefficients are used to linearly predict the necessary correction weight and residual unbalance. If the first two correction weights are not successful at reducing the vibration, this could mean trouble. Typical indications of possible problems include:

- The program predicts that moving the same balance weight by approximately 90° or less will significantly lower the residual vibration. However, when the weight is installed the actual vibration is not going down, and the program continues to rotate the weight around the balance plane (dog chasing its tail).
- Predicted residual vibration levels will not meet acceptable limits.
- Program is calling for excessive balance weight (multiple times API or ISO) or for unusually small weights.
- Reasonably sized trial weights are not affecting the vibration readings (amplitude or phase).
- Different combinations of trial runs do not predict similar results. If there are enough measurement locations, both single and multi-plane calculations could be performed for comparison. Influence coefficients are inconsistent.
- Program is predicting a trial weight location that has already been tried and is known to cause an undesired result.

Of course, if it's 3:00 in the morning you may start wondering if a human error has occurred, such as not writing down the vectors correctly, placing the balance weight at the wrong location, or misapplying the balance program method. Although no one at the plant wants to hear bad news, it is time to face the fact that this machine cannot be balanced in the present condition.

The following six case histories present situations where balancing difficulties were encountered. These examples show how other problems had to be solved prior to achieving acceptable vibration levels though balancing. Hopefully, this will provide some ideas and techniques to help diagnose and solve similar situations in the future.

CASE HISTORY 1 — GAS TURBINE COMPRESSOR SET WITH WORN SPLINE CONNECTION

When in remote parts of the world, proper machining and balancing is critical since there may not be machine shops and high speed balance machines nearby. Located on a platform in the Caspian Sea are nine refurbished 4,500 HP gas turbine driven centrifugal compressors used for gas gathering/transmission. The compressor is coupled through a splined interconnect shaft to the power turbine (PT) which is driven from the exhaust of the gas producer (GP) turbine at a speed of approximately 15,500 RPM. The interconnect shaft is basically a 21″ long spool spacer torque tube with internal gear teeth on each end to transmit the torque. The interconnect slides over geared coupling adaptors on the PT and compressor with mating external gearing. The power turbine is a single stage over-hung rotor supported by two bearings. For machine protection, a velocity transducer was mounted on the power turbine bearing housing in the horizontal direction. The power turbine alarms at 0.7 ips zero-peak and trips at 1.1 ips zero-peak overall.

During commissioning, a few of the power turbines were experiencing sporadic synchronous vibration at 1× running speed during startup and steady-state operation. A typical occurrence would be during startup; the 1× vibration would increase and exceed the trip set point at 97 percent speed.

Upon the next startup, the unit would reach maximum running speed; however, the 1× vibration would increase from approximately 0.5 ips zero-peak to 0.8 ips zero-peak within seconds. The unit was unloaded and dropped to 70 percent speed for several minutes at which time the PT vibration reduced to 0.1 ips zero-peak. The unit was brought back to full speed and load and vibration increased to 0.65 ips zero-peak. After several minutes, the PT vibration increased to 1.0 ips zero-peak.

The alignment on the units was checked and found to be within tolerance. The power turbines were boroscoped and the blades were found to be in good condition.

It was determined that the interconnect shaft and power turbine coupling adaptor were not match marked when initially balanced. Residual unbalance from each piece can add or cancel out depending on the angular orientation. By clocking the pieces 120° during three separate runs, the best position was found. However, there were still some problems which increased the vibration.

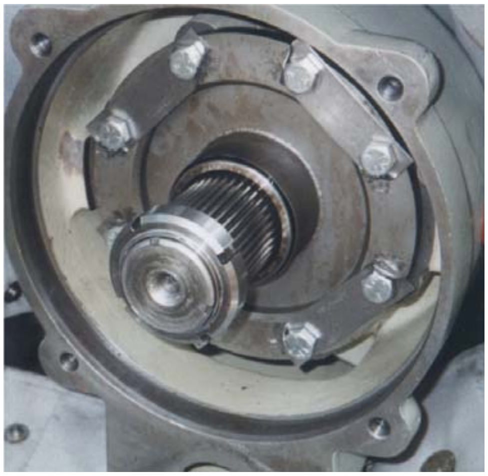

Figure 12: Spline on Power Turbine

In some instances, the vibration was still high at full load and speed. Coupling gear lockup was suspected. The pieces were removed, cleaned, and the gear teeth honed. With some care and proper lubrication, the splined connections worked well and the synchronous vibration was reduced. However, it was not until a new interconnect shaft and PT coupling adaptor was installed that the vibration problem was completely eliminated.

It was suspected that the worn splines allowed the coupling to lockup or allowed the coupling adaptor or interconnect shaft to shift radially due to the centrifugal force, thus causing an unbalance. The problem was corrected without balancing.

CASE HISTORY 2 — THERMAL BOW OF MOTOR DRIVING PIPELINE COMPRESSOR

The operators of a compressor pipeline station reported that during some starts, the motor experienced high vibration. Tests were performed at various operating conditions to diagnose the cause of the high vibrations by monitoring and recording shaft vibration with proximity probes.

During a cold startup and cold shutdown, the vibrations peaked at first lateral critical speed (2,000 RPM) and were acceptable levels. After operating at load (9,450 HP and 4,320 RPM) for two hours to heat soak the rotor, the motor also had acceptable vibration during coastdown though the first critical. After the minimum waiting period of six minutes, the unit was restarted hot. During the restart, it tripped on an overall vibration level of 2.5 mils p-p when the speed reached the first critical speed.

For testing purposes, it was decided to temporarily raise the trip level to 3 mils p-p, so the motor could pass through the critical during a hot startup (Figure 13). After the unit had operated at load (9,100 HP; 4,270 RPM) for approximately 30 minutes, the vibrations reduced and the amplitudes and phases were repeatable with the first loaded run. These results demonstrated a thermal sensitivity related to a hot restart. Since the vibrations at load reduced as the temperature increased, this was not the typical pattern of a thermal bow caused by a "hot spot."

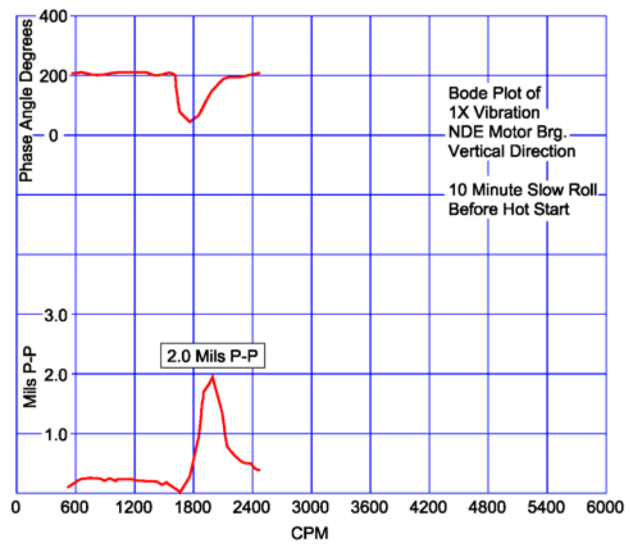

Figure 13: Rotor Vibration During Startup with No Slow Roll Before Hot Start

Since the cooling fans operate continually during the waiting period of six minutes before the hot restart, it was hypothesized that the rotor while at rest was not cooled uniformly, allowing a temporary bow to exist. During startup this bow created an unbalance which caused increased vibrations. After 30 minutes at load, the rotor temperature became uniform and the vibrations decreased and matched the values during the previous loaded run.

As an experiment to minimize the bow caused by possible uneven cooling, the startup procedure was modified to run at idle speed (900 RPM) for ten minutes. During this ten minute period, the vibrations reduced which demonstrated that the thermal bow was also decreased. The unit was ramped from 900 to 2,500 RPM and passed through the first critical. The measured 1× amplitude at the first critical speed was 0.9 mils p-p and was the same as the cold startup (Figure 14). Slow-rolling a heavy rotor is a typical procedure to reduce a "set" of bow in a rotor due to differential cooling.

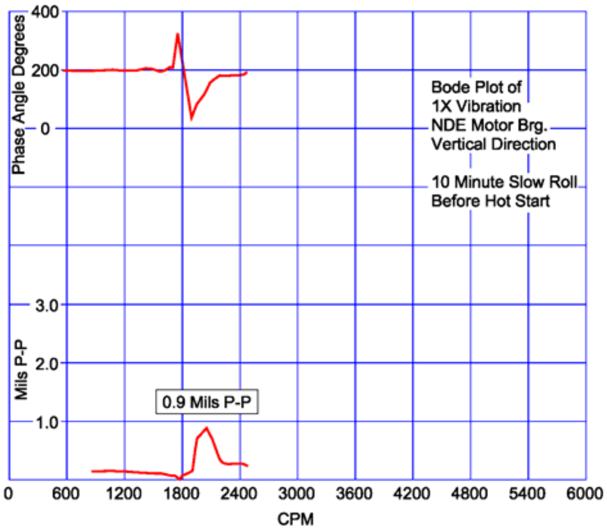

Figure 14: Rotor Vibration During Startup with Ten Minute Slow Roll Before Hot Start

To confirm the success of the ten minute slow-roll period, several more hot restarts were made. Each time the critical speed was passed through below alarm level of 2 mils p-p. The modified startup procedure was acceptable to the station and it was programmed into the computer startup sequence for the motor. Field balancing was not required to reduce the high vibration.

CASE HISTORY 3 — DIESEL ENGINE DRIVING GENERATOR WITH DEFORMED POLES

The synchronous generator shown in Figure 15 was driven by a diesel engine through a bolted flange at the engine flywheel. The single generator bearing (outboard) had excessive vibration at 1× running speed of 450 RPM. The maximum reported generator bearing life was only four months. It was also reported that the engine bearings on the flywheel end had experienced failures.

Figure 15: Generator and Bearing Pedestal

When first arriving on site, the generator outboard bearing was being replaced. An inspection revealed that the old bearing was "wiped" and the babbitt was destroyed. After the new bearing was installed, a vibration survey was conducted.

The horizontal vibration at 1× running speed was 5.6 mils p-p on the engine flywheel end and 6.4 mils p-p at the generator outboard bearing with essentially the same phase angles, which could be indicative of a large static unbalance. For comparison, the 1× vibration amplitudes measured on the other generators were all less than 1 mil p-p.

The bearing housing vibration was recorded during a coastdown with the generator de-energized and plotted in a waterfall format (sometimes called speed raster). A waterfall plot is a series of vibration spectra, which are obtained at specific speed increments and plotted on the same page. The vibration amplitudes at 1× running speed were proportional to the speed squared, again indicating a mechanical unbalance problem and not an electrical problem creating a magnetic force.

Some abnormalities that can cause 1× vibration are listed below:

- Static Unbalance – It exists when the principal axis of inertia is placed parallel to the axis of rotation. This is caused when excess of mass exists on one side of a rotor. An eccentric rotor gives an equivalent condition.
- Bowed Rotor – A shaft condition such that the geometric shaft centerline is not straight.
- Misalignment – Misalignment between the engine and generator outboard bearing. When solid coupling halves are bolted up eccentrically, a crank action is created. For example, a 2 mil angular offset between the flywheel and generator drive flange projected out to the generator outboard bearing could result in a misalignment of approximately 27 mils at the generator outboard bearing. This misalignment would be excessive.
- Soft Foot – Many of the existing shims under the generator outboard bearing feet were replaced with a single shim. All of the anchor bolts were then re-torqued. Thus, soft foot did not appear to be the cause of the vibration.
- Loose Rotor Component – Looseness in the bolted coils on the generator rotor were checked and re-tightened by plant personnel. Therefore, looseness of the coils was eliminated as a possible cause.

The plant requested trim balancing the generator rotor. Generally, generator rotors are machined balanced such that the residual unbalance (amount of unbalance remaining in a rotor after balancing) is less than the API allowable as given in Equation 1. For a total rotor weight of 38,000 lbs and a maximum continuous speed of 450 RPM, the computed allowable residual unbalance is 338 oz-in.

Before proceeding with the trim balance, the generator rotor was inspected for mechanical problems by plant personnel. All of the pole bolts were checked to insure that they were tight. Factory balance weights were found on the inside of the ring as shown in Figure 16. The existing weights on the front and back of the generator totaled approximately 32 lbs. At a radius of 25 inches these weights are equivalent to 12,800 oz-in.

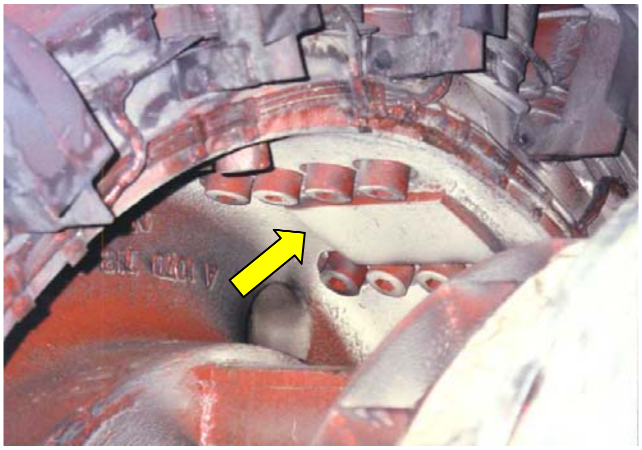

Figure 16: Factory Balance Weight

Trim balancing was attempted on the generator rotor. The factory weights were not altered (original condition) and any additional weights were tack welded to the inside of the core. The trial weights and measured vibration are summarized in Table 4. The location angle of the trial weights is referenced to the factory weights opposite rotation.

Table 4: Generator Outboard Bearing Housing Vibration

Horizontal Direction	**Vertical Direction**
Baseline (Factory Weights Only)	
5.1 mils p-p @ 80°	2.7 mils p-p @ -13°
TW1: 7 lbs @ 112°	
4.9 mils p-p @ 70°	2.6 mils p-p @ -23°
TW2: 21.6 lbs @ 97°	
4.5 mils p-p @ 64°	2.4 mils p-p @ -28°

After the heavy spot was located based on the vibration vectors, a trial weight of 7 lbs (2,800 oz-in) was added opposite the heavy spot. The added mass resulted only in a 4 percent reduction in vibration amplitude in both directions. The trial weight was increased to approximately 22 lbs (8,800 oz-in) in the same quadrant; however, only a 12 percent reduction in vibration occurred in both directions.

It was estimated that as much as 180 lbs would be required to balance the generator. Therefore, the forces producing the high vibration are very large and it would not be practical to reduce the vibration levels by trim balancing the generator rotor so the trial weights were removed.

Conclusions and Recommendations

Due to the limited time on site, the following recommendations were made prior to departure:

- Check alignment to determine if the generator outboard bearing is aligned with the engine. Since the engine and generator were hard-coupled, the bolted flange joint should also be checked for proper make-up.
- Match mark flanges to indicate angular orientation between engine and generator shafts. Decouple the generator and run the engine solo. Check resulting vibration amplitude at engine flywheel.
- Index the existing generator rotor by 180° relative to the engine or exchange the generator rotor with a spare if available and reconnect the flanges. Re-measure the vibration at the generator bearing.
- A check should be performed to determine eccentricity of the generator rotor and straightness of the generator shaft. Shop balancing and/or re-machining may be required.

It was later reported by plant personnel that the pedestal bearing housing was rocking on the foundation. A thorough check was conducted on the prime mover but no significant problem was found. Flywheel bolts were retorqued and very slight movement was observed.

On one occasion after attempting to synchronize the generator, a loud noise was heard and the unit tripped. Two of the generator poles were found to be badly deformed. While replacing the poles it was discovered that the electrical connections were improperly wired.

When the generator was brought back on-line with the new poles, vibration measurements were retaken. The vibration levels were acceptable and no further dynamic balancing on the generator was required. The unit's vibration levels have remained low and no pedestal bearing failures have reoccurred. This confirmed that the unbalance and high vibration was caused by the deformed poles.

CASE HISTORY 4 — OVERHUNG BLOWER WITH LOOSE HUB

The blower experiencing the high vibration was driven by a 200 HP induction motor at 3,590 RPM. The blower has a single overhung impeller with 12 blades (Figure 17). The shaft is supported by two roller bearings bolted to a sole plate, which is grouted to the top of a concrete foundation. A labyrinth seal is located between the impeller and bearings to prevent gas leakage.

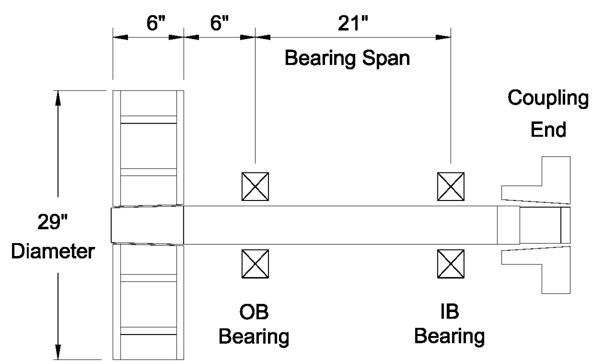

Figure 17: Blower with Overhung Impeller

The blower had been recently overhauled and run approximately three weeks. Plant personnel reported that the vibration levels were initially low, but that subsequent measurements showed a significant increase in vibration. The blower was shutdown and inspected. A “clip-on” balance weight was found inside the machine that appeared to have been thrown from a blade. The balance weight was reattached to the impeller at what was believed to be the original location, but the vibration levels remained high.

Testing

The blower was shutdown so that coupling guard could be removed to install an optical tachometer for a phase reference. Tri-axial accelerometers were mounted on the bearings. The blower was restarted and vibration measurements were taken. Excessive 1× vibration of 6.3 mils p-p was measured in the horizontal direction on the blower outboard (OB) bearing. Figure 18 shows a waterfall plot of the coastdown data.

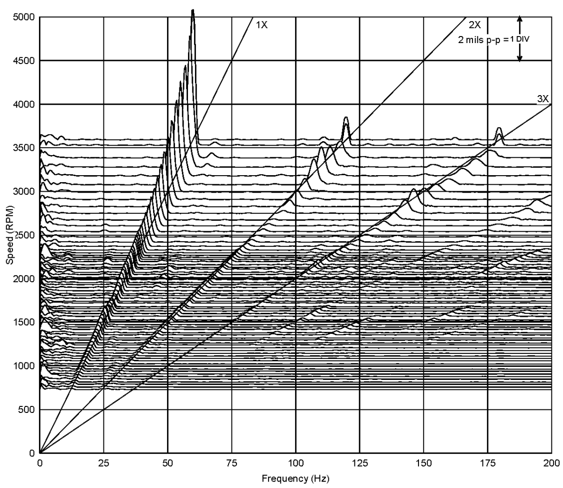

Figure 18: Waterfall Plot of Bearing Vibration During Coastdown

A bode plot during the coastdown is shown in Figure 19. No lateral critical speeds were found near running speed. The phase angle was also steady near the running speed. Indications were that the high vibration levels were caused by unbalance of the blower impeller; however, the amplitude increased at a rate greater than the speed squared. For example, an unbalanced rotor with 1 mil of vibration at 2,000 RPM should have 4 mils at 4,000 RPM. The amplitude tracked with the speed squared until 3,000 RPM and then the amplitude increased at a higher rate (non-linear behavior).

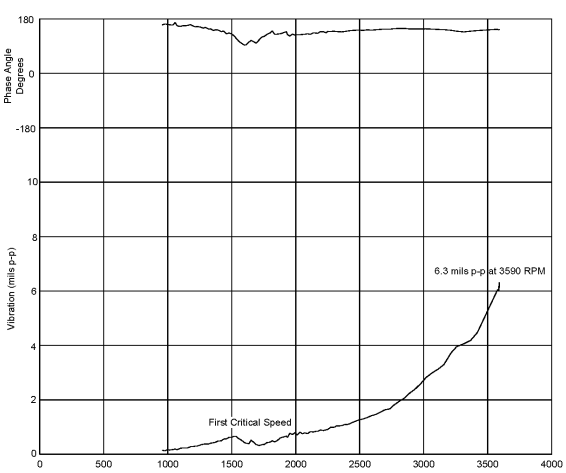

Figure 19: Bode Plot of Bearing Vibration During Coastdown

Due to time constraints and the criticality of the blower, it was decided to trim balance in place and not remove the rotor. Trim balancing was performed through the night. The first run was with the existing weight (290 g-in). For the second run, a baseline was established without any weights. A least-squares balance program was used to calculate the influence coefficients and predict the correction weight. However, data obtained with the correction weight did not yield the predicted results. When this occurs, nonlinearity exists in the system. Additional trial weights were tried in an attempt to lower the vibration into a linear range.

The lowest attainable vibration level after balancing was 3.6 mils p-p on the blower OB bearing in the horizontal direction (a 40 percent reduction). Although the vibration had been improved with field balancing, the levels were still too high to ensure reliable operation until the next scheduled turn-around. The target vibration level was 1 mil p-p at 1× running speed (60 Hz), which is equivalent to a velocity reading of 0.2 ips peak.

An inspection of the blower revealed significant scale build-up on the leading edge of the blades. Therefore, it was decided to pull the rotor, clean the impeller by sand-blasting, and shop balance the assembled rotor at 400 RPM using the Schenck balancing machine. The roller bearings were to be replaced with new spares from the warehouse just in case the existing ones were damaged from the high vibration levels.

When the rotor was removed, hard rubbing was apparent between the shaft and labyrinth seal. This could be due to the high vibration or improper seal installation. This could also cause some nonlinearity in the influence coefficients used for balancing.

There was some difficulty balancing the clean rotor in the balancing machine at the shop because some readings were inconsistent. Balancing continued until a tolerance of 13 g-in was achieved, which met the G3 specification for this rotor. The final shop balance was 310 g-in on the outboard and 240 g-in on the inboard of the impeller, 180° out-of-phase.

Before the blower was reassembled, the motor was run solo. Vibration levels were checked and found to be low, which further confirmed that the source of the high vibration was the blower. The blower was reinstalled and aligned. With the inlet piping spool removed and the case open, impact tests were performed on the impeller and shaft. A horizontal mode was found at 37.5 Hz and a vertical mode was at 42 Hz. These modes have a sufficient separation margin from the 1× running speed of 60 Hz and should not pose a problem to the system.

The blower was started with the inlet blocked off. Run time in the dead head condition was limited due to heat buildup. Several sudden vibration changes were noticed during the run (Figure 20). For approximately 45 seconds, the vibration levels were less than 1 mil p-p, but then suddenly jumped in amplitude.

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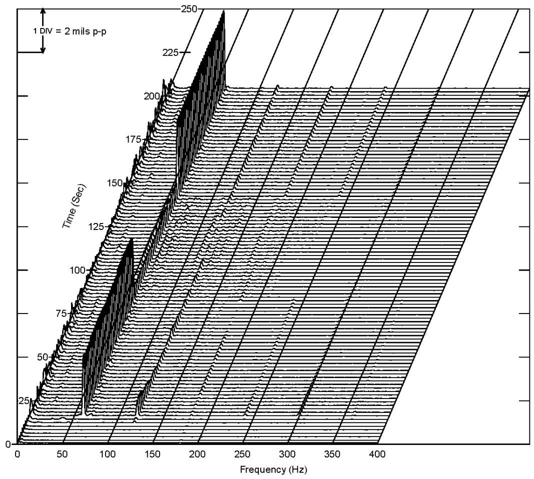

Figure 20: Time Raster of Bearing Housing Vibration

To determine if the seal could be contributing to the problem, it was removed and inspected. Heavy rubbing on one side was again apparent from the first run after the cleanup and rebalance. The labyrinth seal was apparently installed off-center.

During the next run, the blower was operated with the access panels removed and the area was roped off for safety. The vibration levels again shifted during the test indicating that the impeller hub was possibly loose on the shaft. The hub has a 3 mil (0.003″) diametrical clearance on the shaft. The hub and key are held in position with 3 set screws. This type of hub is unacceptable for high speed operation.

Conclusions

1. A review of the available records indicated that this blower has had a history of vibration / balance problems for over seven years.
2. Coastdown and impact data showed that the blower did not have a critical speed problem.
3. Vibration levels were reduced by 40 percent through in-place balancing. However, levels were still too high for reliable operation until the next turn-around. Also, inconsistent results indicated nonlinearity in the blower system.
4. Unusually high correction weights were required for in-place and shop balancing to compensate for the tilting impeller. The centrifugal forces due to these balance weights at operating speed was several hundred pounds, which is more than the total rotor weight of the blower (195 lbs).
5. Several problems were found with the blower: significant scale build-up on the leading edge of the blades, hard rubbing of the labyrinth seal, and hub clearance, which allowed movement of the impeller on the shaft.
6. Recommendations for improving the blower included:
   a. Applying a Teflon coating to reduce scale build-up, which could cause unbalance.
   b. Redesigning the blower using an interference fit for the hub and/or a tapered shaft without a keyway to prevent movement during operation.

A new modified fan rotor was installed a few months later. The new fan wheel had a better fit to the shaft. When this paper was written, the unit had been operating satisfactorily with low vibration levels for over six months since the change out.

More About Balancing Overhung Rotors

Initially, the unbalance distribution was unknown for the preceding rotor. It was assumed that the unbalance was located at the blower impeller. A static unbalance of the impeller would be considered a quasi-static unbalance condition of the entire rotor since it is overhung. As discussed in the previous section, the other two possible unbalance conditions could have been couple unbalance (Figure 4) or dynamic unbalance (Figure 6).

A single-plane balance calculation could be used to correct the static condition, but two-plane balancing is required to correct dynamic unbalance. Normally, static balancing techniques are tried first. There are "rules of thumb" based on impeller width to determine which type of balancing may be required. Alberto (2003) explains that if the ratio of the impeller diameter to width is six or more, then only a static balance should be necessary. The API specifications are more conservative and recommend that all rotating parts be dynamically balanced.

There are other ways to tell which type of unbalance may be occurring. A pure couple unbalance would theoretically produce equal vibration amplitudes at the two bearings. On the other hand, a quasi-static unbalance would create higher vibration at the bearing closest to the impeller. The ratio of the vibration amplitudes at the bearings would be related to the distance from the impeller. Dynamic unbalance would be a combination of the two cases.

For example, a static unbalance of 0.706 oz (20 g) at a radius of 14.5″ on the far impeller end is equivalent to 10.2 oz-in and creates 235 lbs of centrifugal force at a running speed of 3,600 RPM. Using the bearing span and shaft length to create a free body diagram, the resulting bearing forces are computed to be 412 lbs (OB) and 177 lbs (IB), acting in opposite directions.

Couple balancing is typically thought of as placing two weights equal in magnitude, but opposite in phase some distance apart on an impeller. Unbalance of 10.2 oz-in on each end of the impeller, 180° apart creates a moment. The resulting bearing forces of 88 lbs are equal and opposite.

Although the magnitude of the weights used in this example did not change, the resulting forces on the bearings were less for the couple case compared to the static case. A combination of static and couple may be required to reduce the vibration to acceptable levels in some cases where dynamic unbalance exists.

The recommended method to balance an overhung rotor would be to try a single-plane balance to correct the static unbalance first. If the vibration levels at the bearings are still not acceptable, then a couple should be tried next by applying balance weights at each end of the impeller, 180° apart. El-Shafei (2001) discusses balancing overhung rotors using the static-couple method.

CASE HISTORY 5 — CEMENT DUST SEPARATOR WITH STRUCTURAL RESONANCE

Excessive vibration levels were occurring on a dust separator which was installed in the finish mill at a cement plant. The separator was installed on steel beams approximately 60 feet above the ground. The separator is driven by an electric motor through a right angle gear box with a speed reduction of 2:1 (Figure 21). The motor is equipped with a variable frequency drive (VFD) which controls the separator speed up to 400 RPM.

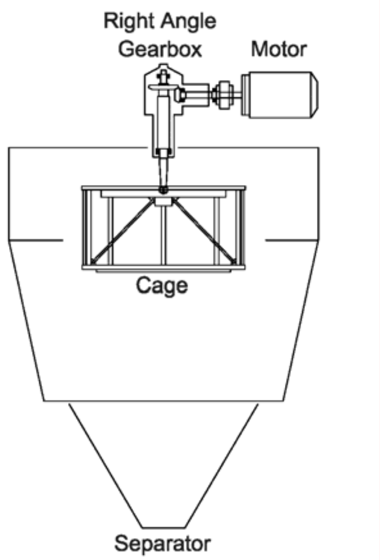

Figure 21: Cement Dust Separator

The motor and gearbox are installed on a pair of wide flange beams, which are mounted on the top of the separator (Figure 22). The separator rejector cage assembly is attached to the end of the vertical gear shaft. Replaceable vertical tubes are located around the periphery of the cage assembly.

Figure 22: Motor and Right-Angle Gearbox

Background

Initially, two other vibration consultants were involved with this machine. They found excessive vibration levels at 1× running speed, but were unable to reduce the vibration by trim balancing the cage.

Additional structural beams were installed in an effort to stiffen the floor under the separator. This reduced the vibration levels by a factor of two. However, attempts to balance the separator were again unsuccessful.

The cage assembly was inspected for mechanical problems which could be contributing to the high vibration levels. Excessive play was found in the cage assembly and signs of wear were noticeable around the air seal at the bottom of the cage. By blueing the inside of the hub, contact was found on only 10 to 15 percent of the tapered surface. The cage appeared to be loose on the shaft.

The original gearbox was replaced with a new gearbox. When the cage was installed on the new gear shaft, the lock nut was tightened using a hammer wrench and the nut was pinned to the shaft to prevent loosening. The fit between the cage and the new gear shaft was reported to have 90 percent contact on the tapered surface.

Balancing was attempted by shifting a 7 pound weight to two different locations over a range of 90° with no change in vibration amplitude or phase angle. The vibration on the gearbox was 80 mils peak-peak at 280 RPM. The vibration data indicated that the entire separator was moving as a rigid body due to vertical vibration of the steel floor beams. It was concluded that the vibration problems were due to structural problems and that the separator could not be balanced without further modifications.

Additional Tests and Analysis Needed

When the author became involved with the project, more inspections and tests were performed to determine the cause(s) of the vibration. This included acquiring vibration data from new measurement locations, performing a lateral critical speed analysis, trim balancing, and conducting impact and shaker tests.

Even with the new gearbox, the cage assembly could still be easily shaken from inside the separator by hand. The assembly was indeed tight on the shaft, but the movement of the cage was due to a rocking of the entire cage assembly and gearbox at the rotor (gear shaft and cage assembly).

An impact test of the cage showed a mechanical natural frequency at 327 CPM, which was just above the maximum operating speed of 300 RPM. A lateral critical speed analysis was performed to help explain if this could be a contributing factor to the high vibration problem.

Critical Speed Analysis

A lateral critical speed analysis was performed using proprietary software, Rotor-E. A computer model of the gear output shaft and the cage assembly was developed using the manufacturer's drawings. The calculated undamped lateral natural frequencies for the first two modes are plotted versus support stiffness in Figure 23. These calculations include the gyroscopic stiffening for the cage running at the normal operating speed 270 RPM.

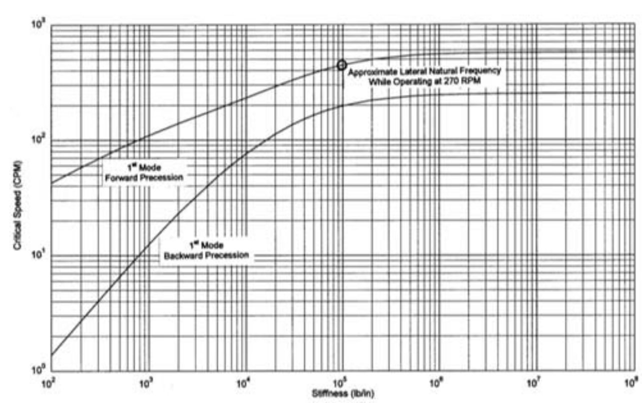

Figure 23: Critical Speed Map

As shown, the lateral natural frequencies are increased as the support stiffnesses are increased. For each mode, there is a forward precession mode and a backward precession mode. The backward precession mode is rarely excited, unless the rotor is unstable and whirls at a non-synchronous frequency. The field data indicated that the cage whirled in the direction of rotation (forward precession) and the vibration was synchronous at the cage running speed. Therefore, for this analysis, the mode of concern is the first forward mode.

Results of the shaker test indicated that the horizontal stiffness at the gear housing was approximately 100,000 lb/in, which would result in an undamped lateral natural frequency of approximately 450 CPM. The calculated response shape is shown in Figure 24.

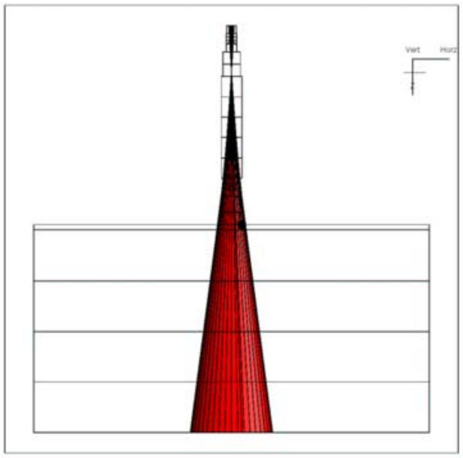

Figure 24: Unbalanced Response of Separator Shaft and Cage

The analysis indicated that there were no rotor lateral natural frequencies in the operating speed range and that the predicted vibrations on the cage would increase as a function of the speed squared due to the unbalance (centrifugal) forces, which would be normal. This meant that the sudden increase in vibration levels near 270 RPM was not due to amplification from a lateral natural frequency of the cage assembly.

Structural Natural Frequencies

Shaker tests were conducted to measure the mechanical natural frequencies of the floor/separator system. The system was excited using a variable speed shaker, which was attached to the side of the gearbox (Figure 25). These tests identified mechanical natural frequencies at 220 CPM, 260 CPM, and 290 CPM.

Figure 25: Shaker Temporarily Mounted to Gearbox

Vibration data obtained during the tests with the shaker indicated that the entire separator was rocking on the floor beams. It was felt that the horizontal and vertical vibration levels could be reduced by installing vertical supports between the over-hung portion of the separator housing and the major floor beams.

In an effort to reduce the rocking motion of the separator, three temporary supports were installed between the overhung section of the separator case and the major floor beams directly below. These vertical supports reduced the gearbox vibration by approximately 50 percent.

Three more vertical supports were installed which resulted in vertical supports on all four sides of the separator. The vibration levels were further reduced with these additional braces. The vibration was approximately 33 percent of that measured during the initial shaker test.

Failed Trim Balancing

After the separator was braced with the temporary supports, trim balancing of the cage was attempted at a speed of 250 RPM, which was below the system natural frequency near 270 CPM. The response of the cage assembly to the balance weights was not predictable, which meant that it would be very difficult to reduce the vibration levels by trim balancing the cage.

Next, trim balancing was continued at a low speed of 150 RPM as previously attempted by the other two vibration consultants. It was again determined that the cage assembly did not respond properly to the balance weights, even at the lower speed.

Additional Measurement Point

Instrumentation was installed to directly measure the vibration levels of the cage. A target for the potentiometers was obtained by attaching a short section of 2 inch diameter shaft to a flat bar which was welded across the lower ring of the cage, Figure 26. The previous shaker data and the critical speed calculations indicated that the vibration levels on the lower ring of the cage could be more than 100 mils p-p when the cage was operating above 200 RPM. Proximity probes typically used to measure shaft vibration could not be used because the linear range of these probes is typically less than 50 mils p-p.

Figure 26: Potentiometers Used for Balancing Cage

As shown, two Spectrol potentiometers were mounted on angle brackets that were welded to the center plate under the cage. This meant that the cage vibration levels were measured relative to the center plate. It was assumed that the vibration levels of the center plate were low compared to the cage which meant that the vibration levels measured with the potentiometers would be approximately equal to the actual cage vibration levels. The potentiometers have a full range of approximately 400 mils p-p (0.4 inch peak-to-peak), which was thought to be adequate for the suspected vibration levels. These potentiometers are not rated for high frequency motion such as this so the stem was greased to reduce the wear.

Vibration data obtained with the potentiometers indicated that the cage vibration levels increased as a function of the speed squared which would be typical for a normal unbalance. The orbits were fairly circular except near 250 RPM where the orbit appeared as a flat ellipse. The maximum vibration levels of 230 mils p-p were measured at a cage speed of 300 RPM.

A Different Approach to Balancing

The previous balance attempts indicated that the cage did not respond in a linear manner, which made it almost impossible to balance the cage using a computer program to compute the balance weights. Therefore, a different technique was used to balance the cage. With the rotor operating at 150 RPM, a large 14 lb balance weight was moved to several locations until the vibration levels significantly changed. After the vibration amplitudes significantly changed, the correction weight location could be computed. When the 14 lb balance weight was installed in the correct location, the vibration levels were reduced.

The final balance weight was 19.6 lbs at 263° as shown in Figure 27. This weight was approximately 1.8 times larger than the original factory weights. The residual unbalance in the cage after the factory balancing can be calculated by vectorially subtracting the factory weights (10.7 lbs at 298°) from the final weights (19.6 lbs at 263°). Using this procedure, the residual unbalance in the cage after the factory balancing was estimated to be 12.5 lbs at the outer radius of the upper ring.

Figure 27: Balance Weights Inside Cage (Looking Toward Top of Cage)

With all the braces permanently installed, the maximum vibration on the gearbox was reduced to 6 mils p-p at 300 RPM. The maximum vibration level of 6 to 7 mils peak-peak on the gearbox is considered to be low for this machine. When the finish mill was in operation, the overall background vibration and noise levels in the building were increased. A visual inspection near the separator indicated that the vibration levels on the separator and the floor deck beams were very low. In fact with the other machines in service, it was difficult to determine if the separator was actually running. The permanent supports shown in Figures 28 and 29 were later painted and appeared to be part of the original design.

Figure 28: Additional Supports

Figure 29: Additional Supports

Allowable Residual Unbalance

The ISO standard provides allowable residual unbalance for various types of rotors. A grade G-6.3 rotor classification includes fans, flywheels, and centrifugal pumps. Since the cage was not a precision rotor, it could also be classified as a grade G-16, which includes automobile drive shafts, parts of crushing and agricultural machinery.

The ISO allowable residual unbalance is given per unit of rotor mass (oz-in/lb). The allowable residual unbalance for this rotor can be determined by multiplying the ISO value by the weight of the shaft and cage assembly, which was 3,300 lbs. The weights were installed on the upper ring at a 30 inch radius. The allowable unbalances for the two grades are shown in Table 5.

Table 5: Allowable Residual Unbalance at 300 RPM

Grade	oz-in/lb	lb-in	lb at 30 inches
G-6.3	0.128	26.4	0.88
G-16	0.32	66.0	2.2

The measured residual unbalance in the cage was 12.5 lbs which was approximately 14 times larger than the allowable levels of 0.88 lbs for the G-6.3 grade, and 5.7 times larger than the allowable level of 2.2 lbs for the G-16 grade.

Conclusions

- The excessive vibration levels on the separator and floor beams were due to the combined effects of excessive residual unbalance in the cage assembly and lack of support stiffness below the separator. Vibration of the separator was reduced to an acceptable level by first installing braces and then by trim balancing the cage assembly.
- Initially, the system behaved nonlinearly, which made it very difficult to trim balance the cage at speed using normal balancing procedures.
- The cage was originally statically balanced at zero speed. This is similar to a static, or bubble balance on a tire. This type of balancing is not adequate to obtain low residual unbalances to meet the G-6.3 specification. The cage should have been dynamically balanced at 300 RPM before it was installed in the separator.
- The footprint of the separator base was too small and contributed to the rocking motion. The vibration levels were significantly reduced by installing supports which was similar to increasing the base dimensions.
- The base was not directly aligned with the major floor beams which made it difficult to properly support the separator. The floor system was too flexible which resulted in excessive vibration levels for a given unbalance force. The floor design could have been improved by locating major beams directly under the separator.
- Several mechanical natural frequencies of the floor/separator system were identified within the cage running speed range. These natural frequencies further amplified the vibration levels for a given shaking force.
- In the design stage, a detailed dynamic analysis should be completed to compute the mechanical natural frequencies of the floor/separator system, and the resulting vibration for a given unbalance condition. There should also be a sufficient separation margin between the operating speed range and the predicted mechanical natural frequencies.

CASE HISTORY 6 — HIGH VIBRATION OF CLUTCH BETWEEN STEAM TURBINE AND GENERATOR

During commissioning of a turbine-generator set, the unit could not operate above 50 percent load due to excessive 1× shaft vibration, as well as clutch housing vibration. A system sketch is shown in Figure 30. The low-pressure (LP) steam turbine is directly coupled to the generator and can produce about 35 MW at 3,600 RPM. For additional power, a high-pressure (HP) steam turbine is connected to the opposite end of the generator through a clutch, which can provide an additional 85 MW when coupled. When both the LP and HP turbines are rotating at the same speed, the clutch will engage and behave as a gear-tooth coupling.

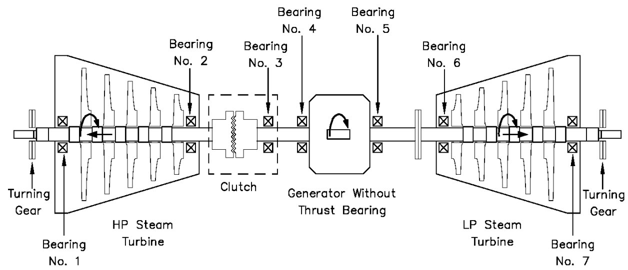

Figure 30: Turbine Generator Drive

With the LP system on-line, the HP turbine would be heat soaked while gradually increasing the HP turbine speed to 3,600 RPM. When the HP and LP turbines were synchronous in speed, the helical splines of the two rotating shafts within the clutch would align and engage. Following the engagement, additional load would be applied to the generator. By design, the clutch would disengage, if the HP rotor was turning slower than the LP rotor.

An interesting fact was that the clutch and bearing vibration amplitudes were different depending on whether or not the clutch was engaged. Diagnostic testing was performed to investigate the cause(s) of the vibration.

Testing

One of the suspected causes of the vibration was a structural natural frequency of the clutch box/foundation near the operating speed. The presence of a structural resonance would make the system more sensitive to unbalance. To determine the structural natural frequency, an air driven mechanical shaker was mounted on the side of the concrete pedestal underneath the clutch box (Figure 31). The shaker produces a rotating force vector with the magnitude proportional to speed squared. The shaft of the shaker was oriented parallel to the turbine shaft so that the shaking forces were produced in the same force plane that would be produced by turbine and/or clutch unbalance.

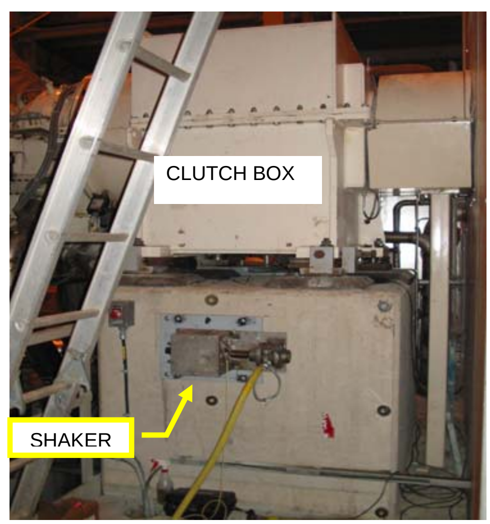

Figure 31: Shaker Mounting

While the unit was down, the shaker speed was varied from 1,000 to 3,850 RPM, which was the maximum speed that could be obtained with the available air supply. The resulting 1× vibration of the clutch box in the horizontal direction was plotted in Figure 32 as a Bode plot versus shaker speed. Note that the measured vibration amplitude was divided by the unbalanced force from the shaker so that the normalized response has units of mils/lb. There was a peak response and accompanying phase shift near 3,700 RPM (62 Hz), which is in close proximity to the running speed of 3,600 RPM. Other response peaks near 1,500 RPM (25 Hz) and 1,700 RPM (28 Hz) were noted, but these peaks were more than 20 percent below the rated speed and are therefore not of concern.

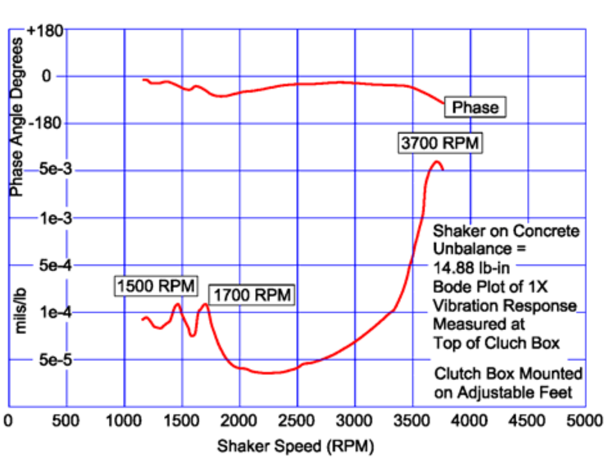

Figure 32: Bode Plot of Normalized Response of Clutch Box in Horizontal Direction during Shaker Test

To characterize the mode shape at 62 Hz, a wire-frame model of the clutch box and foundation was developed. With the shaker speed set at 3,700 RPM, the vibration response was measured at all points in the model. Figure 33 shows that the majority of the flexibility occurred in the mounting of the clutch box. The clutch box was mounted on adjustable "feet" to allow for elevation adjustments.

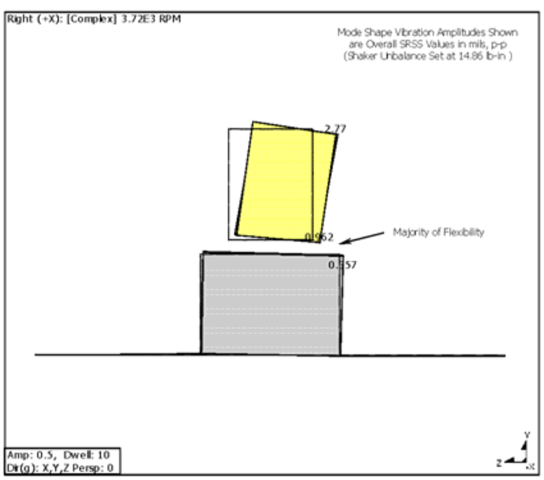

Figure 33: Vibration Mode Shape at 62 Hz

After the first shaker tests, temporary steel shims were wedged between the bottom of the clutch box and the concrete pedestal to determine if any change in the response frequency occurred. As shown in Figure 34, there was a slight increase in the response frequency (blue trace) with the temporary shims. This test indicated that improving the connection stiffness could be beneficial in detuning the resonance.

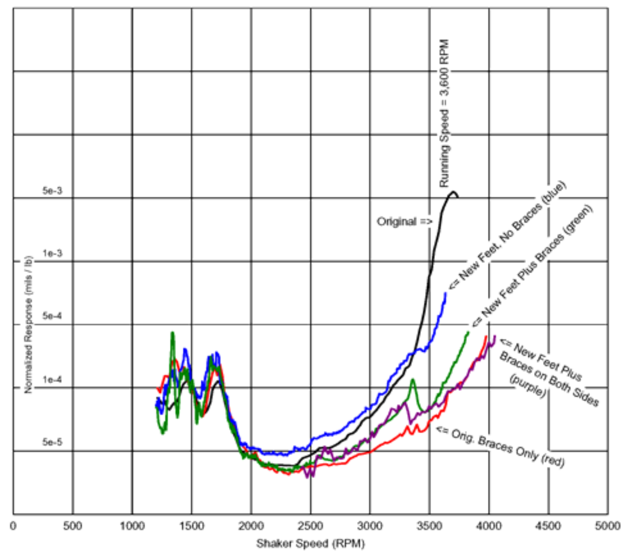

Figure 34: Comparison of Shaker Test Results

Next, diagonal braces from the clutch box case to the concrete pedestal as well as improved wedges between the sole plates and clutch box were installed as shown in Figure 35. The shaker tests were repeated. A comparison between the original (black trace) and modified frequency response (green trace) in Figure 34 shows a significant increase in the structural natural frequency. The exact frequency could not be identified due to the speed limitation of the shaker. Based upon the results of the shaker tests, the vibration reduction at 3,600 RPM was a factor of 2.5 with the improved mounting and diagonal braces to the clutch box.

Figure 35: Clutch Box Diagonal Bracing

Further tests revealed that the clutch box and shaft vibration levels were sensitive to clutch engagement. The clutch between the generator and HP turbine would engage in an arbitrary angular position once the two shafts were synchronized at 3,600 RPM. Any residual unbalance in the two clutch halves will be vectorially combined and the magnitude of the vibration will vary with the engagement angle. In order to monitor the clutch engagement position, the two key phasors (HP and LP) were plotted during one revolution as shown in Figure 36. The phase lag of the HP key phasor relative to the LP key phasor was defined as the engagement angle (or "synch position") for the purposes of the tests.

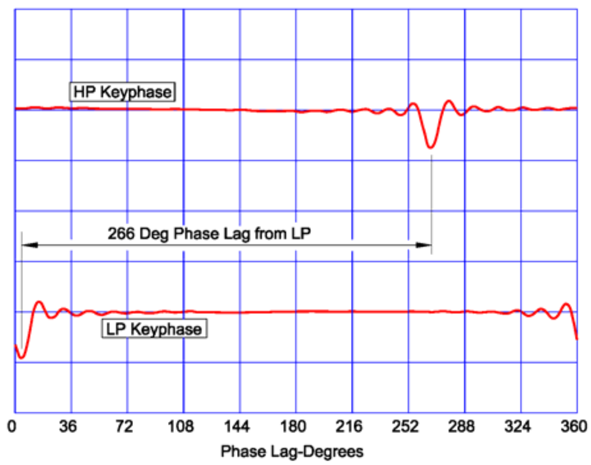

Figure 36: HP Turbine to LP Turbine Phase Lag

The "sweet spot" with minimum vibration was defined with a minimum of three different engagement positions roughly equally spaced between 0 to 360 degrees. From these data, the plot shown in Figure 37 was developed. This allowed for the determination of which range of synchronizing positions would keep the vibration levels below the clutch manufacturer's vibration allowable.

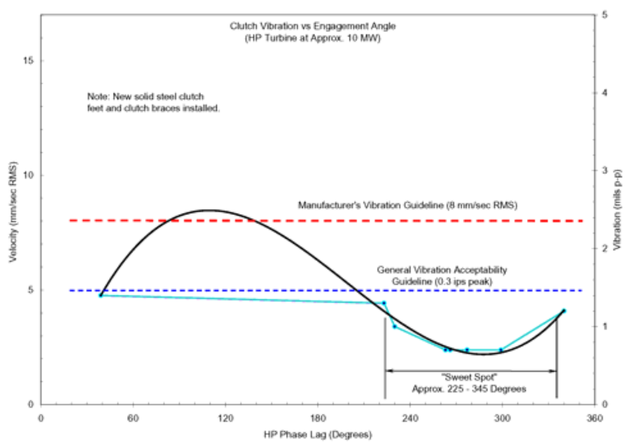

Figure 37: Clutch Vibration vs. Engagement Angle

The cogeneration plant continues to use the LP and HP key phase signals for synchronization and engagement of the clutch to minimize unbalance and vibration of the system.

SUMMARY AND CONCLUSIONS

In this tutorial, unbalance was defined and the four common types were discussed: static, couple, quasi-static, and dynamic. The rotor response depends on the unbalance condition as well as the lateral natural frequencies and mode shapes. The phase relation between the unbalance and vibration vectors shifts based on how close the operating speed is to a critical speed.

Four different balancing methods were reviewed. The simplest procedure was the vector method which was a graphic technique for single-plane balancing. Without phase data, the four-run method can be used. For more complicated rotors running at higher speeds, the modal or influence coefficient methods could be used for multi-plane balancing.

There are many balance criteria available. The proper balance grade should be selected depending on the rotor classification. For turbomachinery in the petro-chemical industry, the API standards are used to specify the allowable residual unbalance and vibration levels at maximum continuous speed. The 10 percent force method is a rule of thumb that could be used to size an initial trial weight when other information is unavailable.

General problems that can affect balancing were discussed. Recognizing these conditions before attempting to balance a machine is necessary for success. Next, tasks were given that should be performed before balancing machinery in the field. Some guidelines were also given on how to avoid potential problems when using the influence coefficient method of balancing.

Six different case histories were presented. Lessons learned from these difficult balance jobs include:

- Checking spline connections for wear and lockup. Match marking balanced parts so that they do not get indexed incorrectly.
- Preventing thermal bows in motors by slow rolling before hot startup. Cooling fans could also be designed to provide more even flow around the rotor.
- If an unreasonably high amount of weight is necessary to correct a balance problem, careful mechanical checks should be made. Damaged or loose parts may be responsible for the unbalanced condition.
- Hub designs for lower speed machines, may not be appropriate for operation at 3,600 RPM. Proper mounting and inference fit should be used. Looseness can allow tilting of an impeller and significant unbalance and vibration.
- Structural resonances can greatly increase the sensitivity of a machine to unbalance. Dirty conditions can contribute to ongoing vibration and maintenance problems. Operation at critical speeds and structural natural frequencies should be avoided by an appropriate separation margin. Performing rotordynamic and finite element analyses in the design stage could also help prevent this problem.
- Rotating parts should be connected with the same angular orientation as when originally balanced. Changing this orientation could cause the residual unbalance of each part to combine differently, possibly increasing the overall unbalance of the machine.

Balancing can be very challenging under some circumstances. Once certain conditions have been identified and corrected, trim balancing should be much easier and in some cases may not even be required to solve the vibration problem.

NOMENCLATURE

A	vibration level, mils p-p
CG	center of gravity
cpm	cycles per minute
F	centrifugal force, lbs
H	high spot
Hz	Hertz, cycles per second
ips	inches per second
m	mass
mil	one thousandth of an inch (0.001″)
N	speed, RPM
O	journal axis
p-p	peak-to-peak
rpm	revolutions per minute
S	spin axis
U	unbalance, oz-in
W	weight, lbs
ε	eccentricity, in
ω	angular velocity, rad/sec

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